Graduate Program in Mathematics
University of Virginia

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The Mathematics Department administers programs leading to Master of Arts, Master of Science, and Doctor of Philosophy degrees in mathematics. These programs provide diverse opportunities for advanced study and research in algebra, analysis, topology, mathematical physics, history of mathematics, and probability. The mathematics department has about 29 regular faculty members, nearly a dozen visitors and postdocs, and 41 graduate students, creating a close-knit community of faculty and students.

The mathematics department is classified as a Group I department by the American Mathematical Society, the smallest such department at a public university in the country. All permanent mathematics faculty are internationally prominent in their research specialties. The faculty are often invited as visitors overseas, a number have been involved in organizing major national conferences, and many serve on the editorial boards of major journals. Most have been awardees of prestigious awards, including Sloan, NSF Postdoctoral, NSF CAREER, AMS, and Guggenheim Fellowships. In recent years, eight of our faculty have written books and papers chosen for a 'Featured Review' by Math Reviews, deemed by the AMS to be the among the best 100 publications of the year in the world of mathematics. In addition, two of our faculty are included in the ISI Highly Cited list of the 50 most cited mathematicians worldwide.

Our Whyburn Instructor postdoctoral program has in recent years brought us some of the world's best new Ph.D.'s in areas including probability, algebraic topology, algebraic coding theory, mathematical physics, and pde's.

Numerous subfields of mathematics are represented in the research of the faculty. In algebra, the main areas are linear and arithmetic groups and associated structures, nonassociative algebra and geometry, and representation theory. In analysis, the focus is on differential equations and applied mathematics, operator theory, operator algebras, and function theory, probability, and mathematical physics. In topology, algebraic topology and geometric topology are both studied. In our history of mathematics program, emphasis is on the nineteenth and twentieth centuries.

Besides a series of weekly colloquium lectures, there are also research seminars in operator theory/operator algebras, mathematical physics, algebraic topology, geometric topology, differential equations, algebra, probability, and history of mathematics. Each year, our Institute of Mathematical Science provides the structure for a series of seminars and conferences in a particular area of emphasis. The graduate student seminar serves as a meeting place for graduate students and junior faculty to talk about mathematics, and fosters an unintimidating atmosphere for discussion.

The department and library are located in Kerchof Hall. The library houses a research collection primarily for the use of mathematics faculty and graduate students, with about 34,000 volumes and subscriptions to over 185 research journals. The mathematics department has a well-equipped computing laboratory available to faculty and graduate students, as well as a computer in every office.

Degree Programs and Requirements

The Master of Arts and Master of Science Degrees are normally completed within two years, during which time the student fulfills course requirements and passes a final examination. In some cases, these degrees can be completed in one calendar year (two semesters and a summer session). The M.A. and M.S. programs differ mainly in course requirements. The M.S. degree in mathematics requires specific courses in algebra, analysis, and topology. In contrast, the course requirements for the M.A. degree in mathematics are flexible and based on individual needs. The candidate for the Master's degree in mathematics has two options, one requiring an expository paper for a thesis, the other substituting additional coursework in place of a thesis.

The Doctor of Philosophy degree is normally completed within five years. Candidates for the Ph.D. must fulfill certain course requirements and examinations beyond the Master's level. The most important addition is the Ph.D. dissertation, which is based on original research performed under the supervision of a faculty member.

In all programs, students may take approved courses from other departments.

There is a teaching requirement for advanced degree programs in mathematics: all full-time graduate students receiving financial aid are required, as part of their graduate program, to gain teaching experience by assisting in the instruction of undergraduate courses. Normally the student simultaneously fulfills the teaching requirement and obtains financial aid by teaching or directing discussion sections for undergraduate courses. Teaching is evaluated, and a good record is an asset to graduates seeking academic employment. Some students obtain their financial support by serving as computer assistants (CA's) after receiving appropriate training (provided by the department).

Requirements for the Doctor of Philosophy Degree in Mathematics

Course requirements: 72 hours of coursework at the 5000 level or above (which may include 18 hours of non-topical research: MATH 8998, 8999, 9998, 9999). A student must do satisfactory work in two semesters of analysis (MATH 7310, 7340), algebra (MATH 7751, 7752), and topology (MATH 5770, 7800), or the equivalent.

Examinations: Passing grades on two general examinations, chosen from analysis, algebra, and topology, and satisfactory performance on the Second-Year Proficiency Examination.

General examinations: The General Examinations, set and graded by the graduate committee, are written exercises. They test whether a student has the inventiveness and command of material basic to the pursuit of a Ph.D. degree. The examinations cover the following material: Algebra (MATH 7751 and 7752), Analysis (MATH 7310 and one additional course selected from 7340, 7360, and 7250), and Topology (MATH 5770 and 7800). They are offered two times a year, in January and August. A student is expected to take these examinations at the first opportunity following completion of the corresponding two-course sequences, that is, at the beginning of the second year of graduate study. In any case, these exams must be satisfactorily completed by January of the student's third year. A student is allowed at most a total of four sittings for exams (taking one exam is one sitting).

Second-year proficiency examinations: Students take an informal oral examination on material from two or three selected second-year courses. The exact content of the exam is determined by a panel of faculty members in consultation with the student. Its purpose is to gauge the student's readiness to begin carrying out research in the student's chosen area. It is normally taken in May of the second year. If any deficiencies are noted, the examining panel will make recommendations on how to fix the detected problems and meet again with the student in August.

Language: Facility in reading mathematical literature in one language (French, German, Italian, Russian, or a substitute acceptable to the department), as demonstrated by an examination administered by the department, in which the students are required to translate passages from mathematical works in the given languages. The language requirement should be satisfied by the end of the student's fourth year, or by the date of the Ph.D. defense, whichever comes first.

Students pursuing research in the history of mathematics are required to pass a written translation examination in two foreign languages, typically French and German, although the languages required will depend on the student's research interests. These language requirements should generally be satisfied by the end of the student's third year in order to enable the student to do primary source reading in the pertinent languages.

History of Mathematics Program: The requirements for the Ph.D. in the history of mathematics are the same as for the Ph.D. in mathematics, with the exception of the additional language requirements described above. Also, students entering the program must exhibit a strong reading comprehension in two foreign languages, preferably French and German. Students interested in pursuing a Ph.D. in history of mathematics should contact Professor Karen Parshall before submitting their application.

Dissertation and defense: Written under the supervision of the major advisor, the Ph.D. dissertation must contain original contributions to the field of mathematics. The main results of the dissertation are presented at a public oral defense. A committee consisting of the major advisor and three other faculty members (two from within the department and one from outside) must approve the dissertation and defense in order for the dissertation to be considered accepted by the faculty.

Time limit: The Ph.D. may be completed in as few as three years but must be completed within seven years.

Requirements for the Master's Degrees in Mathematics

Course requirements: (a) Thesis option: 24 credit hours of approved courses at the 5000 level or above (some courses from other departments can count toward the 24 hours). (b) Non-thesis option: 30 hours of courses at the 5000 level or above (no reading or research courses), which must include MATH 5310, 5330 (or replacements from among 7310, 7320, 7340) and MATH 5651, 5652 (or replacements from 7751, 7752), and cannot include more than 9 hours from other departments. The following courses will not be allowed to be used toward the 30-hour M.A. requirement: MATH 5100, 5210, 5340, 5654, and, at most, one of either MATH 5010 or 5030.

Thesis (option (a) only): The Master's thesis is an expository paper written under the supervision of a faculty advisor.

Examinations: For the M.S. degree, students must pass one General Examination chosen from the areas of analysis, algebra, and topology. For the M.A. degree, the student may satisfy the examination requirement by either a passing grade on one General Examination or a passing grade on a Final Masters Examination, the scope of which must be agreed upon in advance by the examiners in consultation with the student. Masters candidates must be registered students at the time of the examination and must complete the degree requirements within three years of passing the examination.

Time Limit: Minimum one year, maximum five years.

The requirements for the M.S. degree are the same as for the M.A. degree, except that the program must include MATH 7310, 7340, MATH 7751, 7752, and MATH 5770, 7800 (or higher substitutes).

Transfers

With the approval of the Department and the Dean of the Graduate School, up to one session of the required three sessions of graduate work may be completed at another graduate school.

Academic Life

Beginning graduate students are advised by the Graduate Advisor. Usually in the second year students acquire a major professor who does all subsequent advising. The responsibility rests with the student to contact a prospective major professor. The advisor approves course selections, monitors progress, and generally oversees the student's program of study. Satisfactory progress is usually measured by a grade of at least B+ in all courses.

The following describes a core program commonly taken by prospective M.S. or Ph.D. students in mathematics during the first year:

Fall Semester
MATH 5770: General Topology
Topological spaces and continuous functions; product and quotient topologies; compactness and connectedness; separation and metrization.

MATH 7310: Real Analysis and Linear Spaces I
Introduction to measure and integration theory.

MATH 7751: Algebra I
Detailed study of groups, rings, fields, modules, and multilinear algebra.

Spring Semester
MATH 7340: Complex Analysis I
Fundamental theorems of analytic function theory.

MATH 7752: Algebra II
Further topics in groups, rings, fields, and multilinear algebra.

MATH 7800: Algebraic Topology I
The fundamental group and covering spaces, Van Kampen theorem, and applications to group theory. Simplicial, cellular, and singular homology; Eilenberg-Steenrod axioms; categories and functors.

Students with advanced preparation or specialized interests may, with permission of the graduate advisor, construct a suitably modified program. Students needing additional preparation are advised to take other courses, such as a 5000-level analysis or algebra sequence, before taking the 7000-level courses:

MATH 5310: Introduction to Real Analysis
Basic topology of Euclidean spaces, continuity and differentiation of functions on a single variable, Riemann-Stieltjes integration, and convergence of sequences and series.

MATH 5330: Advanced Multivariate Calculus
Differential and integral calculus in Euclidean spaces, implicit and inverse function theorems, differential forms and Stokes' theorem.

MATH 5651: Advanced Linear Algebra
This course includes a systematic review of the material usually considered in MATH 3351 such as matrices, determinants, systems of linear equations, vector spaces, and linear operators. However, these concepts will be developed over general fields and more theoretical aspects will be emphasized. The centerpiece of the course is the theory of canonical forms, including the Jordan canonical form and the rational canonical form. Another important topic is general bilinear forms on vector spaces. Time permitting, some applications of linear algebra in differential equations, probability, etc., are considered.

MATH 5652: Introduction to Abstract Algebra
Focuses on structural properties of basic algebraic systems such as groups, rings and fields. A special emphasis is made on polynomials in one and several variables, including irreducible polynomials, unique factorization and symmetric polynomials. Time permitting, such topics as group representations or algebras over a field may be included.

In the second year and beyond, students choose from more specialized courses. Ph.D. students past the third year are mainly involved in seminars and independent research. In seminars, students have the opportunity to lecture on published work or their own research, gaining experience in exposition of advanced mathematical topics. For descriptions of the remaining graduate courses, see the Graduate School's catalogue.

Our standard Ph.D. program for a typical student is roughly as follows:

First academic year: While supported by a teaching assistantship (e.g., running two hours per week of discussion sections of calculus), students take core courses in algebra, analysis, and topology, providing the foundation for all further graduate work.

Second academic year: Teaching responsibility usually involves four contact hours per week, typically running one discussion hour and meeting one's own class of a 1000-level math course for three hours. Students continue to take a range of basic courses, but chosen with potential areas of specialization in mind. General Examinations preferably are passed either just before this year starts, or as soon as possible thereafter, and the Second-Year Proficiency Examination is taken at the end of the second academic year.

Third academic year: Students should be integrating themselves into the research life of the department through advanced courses and participation in seminars. Often students are doing independent reading toward acquiring the specialized background needed for doing research, guided by a potential thesis advisor.

Fourth academic year and beyond: Students should be reading the literature related to a potential thesis topic, followed by jumping into dissertation research. (We view five or six years as the normal time needed to complete graduate work.)

Guide to Research Areas

The Mathematics Department at the University of Virginia offers graduate students the opportunity to do research in a wide range of specialties. To help students with the daunting task of planning their multi-year program, in this guide we describe standard routes through the main research areas that students can currently pursue. We expect most students to follow one of these paths.

For convenience, we have grouped these within Programs in Algebra, Analysis, Topology, and the History of Mathematics. However, it is to be emphasized that there is much interaction between these, and a course of study might easily fall between areas. Furthermore, research areas undergo constant change due to changing faculty and student interests, and to new faculty joining the department. Finally, there are a number of possible courses of study not listed here that may involve collaboration with faculty from other departments.

1 Graduate Program in Algebra

Faculty: P. Abramenko, M. Ershov, K. McCrimmon, B. Parshall, A. Rapinchuk, L. Scott, and W. Wang

Graduate research in algebra is organized into the following areas:

- Linear and Arithmetic Groups and Associated Structures,
- Nonassociative Algebra and Geometry, and
- Representation Theory.

1.1 Core Courses and Requirements in Algebra

The following is the list of basic, graduate courses in algebra prerequisite for students intending to pursue studies in algebra:

- **First Year**
  First Semester: MATH 7751 Algebra I
  Second Semester: MATH 7752 Algebra II

- **Second Year**
  First Semester: MATH 7753 Algebra III (algebras over a field, Artin-Wedderburn theory and the Jacobson radical, applications to representation theory) and MATH 9950 Algebra Seminar.
  Second Semester: MATH 7754 Algebra IV (topics in algebra) and MATH 9950 Algebra Seminar.

  Students also take one additional algebra course in one of the two semesters.

Besides taking the sequence MATH 7751, 7752, 7753, 7754 in their first two years, students with interests in algebra are required—within the first two years—to take at least one additional course in the specialized area of algebra which they expect to follow. The additional course(s) may be an independent reading course taken under the supervision of a faculty member.

General Exam
The General Exam in Algebra is based on the material of MATH 7751, 7752.

Research Seminar
Algebra students are also required to take and to participate actively in MATH 9950 (Algebra Seminar) every semester after the first year. In the *first semester of the second year*, students contemplating working in algebra should contact a faculty member regarding a topic for a literature survey and, during the *second semester*, give a short expository talk in the Algebra Seminar.

Second-Year Proficiency Exam
In the second year, students take the Second-Year Proficiency Exam, which, in algebra, consists of a conversation with a panel of faculty members on the material of two or three second-year algebra courses taken by the student, and on the bibliographical research done by the student and presented in MATH 9950.

1.2 Linear and Arithmetic Groups and Associated Structures

Faculty: P. Abramenko, M. Ershov, and A. Rapinchuk

Research in this area focuses on structural, combinatorial and homological properties of linear groups over general rings with a special emphasis on arithmetic rings (i.e., the rings of S-integers in global fields). Topics include the normal subgroup structure of the groups of rational points of algebraic groups and of their important subgroups, finiteness properties of arithmetic groups in positive characteristic, the rigidity of representations of finitely generated groups and building theory, in particular, group actions on spherical, affine, and twin buildings. The work in this area requires methods of the theory of algebraic groups, algebraic number theory, homological algebra, and combinatorial geometry/topology.

Recommended Advanced Courses
MATH 7600 Homological Algebra, MATH 8510 Group Theory, MATH 8600 Commutative Algebra, MATH 8620 Algebraic Geometry, and special topics courses (or reading courses) in algebraic groups, arithmetic groups, geometric group theory, algebraic number theory, and building theory.

1.3 Nonassociative Algebra and Geometry

Faculty: K. McCrimmon

Research in this area investigates the nature of certain nonassociative algebraic systems which are of fundamental importance in the study of geometry, differential geometry, Lie and algebraic groups, combinatorics, and other areas. Topics include infinite-dimensional Lie algebras (especially Kac-Moody algebras), applications of Lie and Jordan algebras to projective geometry (Barbillian planes, n-gon geometries), and the structure of Jordan superalgebras, triples, and pairs. Research in this area requires familiarity with many aspects of ring theory and multilinear algebra, as well as the theory of Lie, Jordan, and alternative algebraic systems.

Recommended Advanced Courses
MATH 8855 Theory of Algebras, MATH 8710 Lie Algebras, and special topics courses (or reading courses) in any of the areas listed in the general description above, as offered.

1.4 Representation Theory

Faculty: B. Parshall, L. Scott, and W. Wang

Representation theory deals with representations of algebraic and associated finite groups, associative and Lie algebras, and connections with algebraic geometry and mathematical physics. Topics include representations of reductive algebraic groups in positive characteristic with applications to finite groups of Lie type, quantum groups and Hecke algebras, quasi-hereditary algebras and vertex algebras. This work used methods from the theory of algebraic groups and algebraic geometry, Lie algebras, and homological algebra.

Recommended Advanced Courses
MATH 7600 Homological Algebra, MATH 8510 Group Theory, MATH 8520 Representation Theory, MATH 8620 Algebraic Geometry, MATH 8700 Lie Groups, MATH 8710 Lie Algebras, and special topics courses (or reading courses) in algebraic groups, Kac-Moody algebras, symmetric groups and their representations, Hecke algebras and quantum groups.

2 Graduate Program in Analysis

Faculty: A. Abdesselam, C. Gromoll, Z. Grujic, I. Herbst, J. Imbrie, T. Kriete, I. Lasiecka, B. MacCluer, T. Melcher, D. Ramirez, D. Sherman, L. Thomas, and R. Triggiani

Graduate research in analysis is organized into the following areas:

- Partial Differential Equations and Applied Mathematics,
- Operator Theory, Operator Algebras, and Function Theory, and
- Mathematical Physics.

2.1 Core Courses and Requirements in Analysis

Students intending to do research in some area of analysis must take the following courses:

- **First Year**
  MATH 7310 Real Analysis and Linear Spaces I, MATH 7340 Complex Analysis I.
- **Second Year**
  First Semester: MATH 7410 Functional Analysis I.

Additional courses and requirements depend on the specific research area selected by a student, as specified below.

General Exam
The General Exam in Analysis has two parts: the first part based on material from MATH 7310, and the second part based on material from one of MATH 7340, MATH 7250, and MATH 7360. For students planning to work in analysis, the second part of the general exam should fit with the area they are pursuing.

Research Seminars
Students in analysis in the second year and beyond are expected to participate in one of the analysis research seminars. These seminars are an important component of the graduate program, and are student-oriented. They aim to expand upon material covered in the various courses and to prepare students for independent reading of research papers.

In the *first semester of the second year*, students contemplating working in an area of analysis should contact a faculty member regarding a topic for a literature survey, and, during the *second semester*, give a short talk in the appropriate seminar.

Second-Year Proficiency Exam
Also in the second year, students take the Second-Year Proficiency Exam, which, in analysis, consists of a conversation with a panel of faculty members on the material from two or three second-year analysis courses, and and on the bibliographical research done by the student for their seminar presentation.

2.2 Differential Equations and Related Applied Mathematics

Faculty: Z. Grujic, I. Lasiecka, and R. Triggiani

This area focuses on the qualitative study of solutions of differential equations: ordinary differential equations (ODE's) as well as partial differential equations (PDE's), both linear and nonlinear. Particular emphasis is placed on equations arising in mathematical physics and related areas of applied mathematics. Topics of study include fluid dynamics, linear and nonlinear elasticity and wave propagation, harmonic analysis, dynamical systems, and control theory. The mathematical methods used draw from real and complex analysis, functional analysis, harmonic analysis, ordinary and partial differential equations, basic differential geometry, and probability.

Area Coursework
Besides the three common core analysis courses, students should also take MATH 7250 (Ordinary Differential Equations I) as soon as possible. In the second year, students should take MATH 8250 Partial Differential Equations I, and, if possible, MATH 7320 (Real Analysis II) and MATH 7420 (Functional Analysis II). They should also begin attending MATH 9250 (Differential Equations Seminar). Students should try to take the General Exam in analysis on MATH 7310 and MATH 7250; however, the combination of MATH 7310/7340 is also acceptable.

Recommended Advanced Courses
MATH 726 (Ordinary Differential Equations II) and MATH 826 (Partial Differential Equations II), possibly in reading course format. Also MATH 7360 (Probability), MATH 8310 (Operator Theory I), MATH 8360 (Stochastic Differential Equations), MATH 7450 (Mathematical Physics), and MATH 8720 (Differential Geometry).

2.3 Mathematical Physics

Faculty: A. Abdesselam, I. Herbst, J. Imbrie, and L. Thomas

Our research in mathematical physics is concerned with the spectral and scattering theory for Schrödinger operators in quantum mechanics, equilibrium and non-equilibrium statistical mechanics, and topics in classical mechanics. The mathematical methods needed include: real analysis—measure theory and integration; functional analysis—for example, operators in Hilbert spaces; Fourier analysis; partial differential equations; and some basic probability theory.

Area Coursework

Besides the three common core analysis courses, students should also take as soon as possible MATH 7250 (Ordinary Differential Equations I). In the second year, students should take, if possible, MATH 7320 (Real Analysis II), MATH 7420 (Functional Analysis II), MATH 7450 (Mathematical Physics), and participate in MATH 9450 (Mathematical Physics Seminar). Math physics students should take their General Exam in analysis on MATH 7310 and MATH 7340.

Recommended Advanced Courses

MATH 8250 (Partial Differential Equations), MATH 7360 (Probability), MATH 8450 (Topics in Mathematical Physics) as appropriate to the student's thesis work.

2.4 Operator Theory, Function Theory, and Operator Algebras

Faculty: T. Kriete, B. MacCluer, and D. Sherman

Our research on Hilbert space operators draws broadly from functional analysis and has two main (interrelated) strands. One is rooted in complex function theory and concerns composition, Toeplitz, and other operators on spaces of analytic functions. The other studies algebraic structures of operators: von Neumann algebras, C^*-algebras, operator spaces, and noncommutative function spaces.

Track Coursework

Area coursework: Besides the three common core analysis courses, students should take MATH 8310 (Operator Theory) or MATH 8300 (Function Theory) by the end of their second year. Also in the second year students should take, if available, one or both of MATH 7320 (Real Analysis II) and MATH 7350 (Complex Analysis II), as well as MATH 7420 (Functional Analysis II). Beginning in the second year, students should participate in MATH 9310 (Operator Theory Seminar). Students should take their Ph.D. General Exam in analysis in MATH 7310 and MATH 7340.

Recommended Advanced Courses

MATH 7250 (Ordinary Differential Equations), MATH 7360 (Probability Theory), MATH 7450 (Mathematical Physics), MATH 8250 (Ordinary Differential Equations), MATH 8320 (Operator Theory II), MATH 8400 (Harmonic Analysis).

2.5 Probability and Related Applied Mathematics

Faculty: C. Gromoll and T. Melcher

Probability is the mathematical theory of random events and random variables. Areas of particular interest to faculty include central limit theorems, Malliavin calculus, stochastic differential equations, Markov and Lévy processes, stochastic networks, measure-valued processes, and applications to operations research and mathematical biology.

Track Coursework

Area coursework: Besides the three common core analysis courses, students should take MATH 7360 (Probability Theory I) and MATH 7370 (Probability Theory II) as soon as possible. These are typically offered in the Spring and Fall respectively, so that students can begin in the Spring of their first year after taking 7310. In the second year, students should take MATH 8360 (Stochastic Calculus and Differential Equations). Students should also participate in MATH 9360 (Probability Seminar). Students should take their Ph.D. General Exam in analysis in MATH 7310 and MATH 7360.

Recommended Advanced Courses

MATH 8370 (Topics in Probability), MATH 7320 (Real Analysis II), MATH 7420 (Functional Analysis II), MATH 8250 (Partial Differential Equations), MATH 8310 (Operator Theory), MATH 8720 (Differential Geometry) as appropriate to the student's thesis work.

3 Graduate Program in Topology

Faculty: G. Arone, M. Hill, V. Krushkal, N. Kuhn, and T. Mark

Graduate research in topology is organized into the following areas:

- Algebaic Topology,
- Geometric Topology.

We must emphasize that these areas have a lot in common, so the subdivision into “algebraic” and “geometric” is not always precise.

3.1 Core Courses and Requirements in Topology

The following is the list of basic, graduate courses prerequisite for students intending to pursue research in topology:

- **First Year**
  First Semester: MATH 5770 General Topology (point-set topology).
  Second Semester: MATH 7800 Algebraic Topology I. (fundamental group and covering spaces, singular and simplicial homology).
- **Second Year**
  First Semester: MATH 7810 Algebraic Topology II (cohomology, Poincaré duality) and MATH 9800 Topology Seminar.
  Second Semester: MATH 7820 Differential Topology (manifolds, characteristic classes, De Rham cohomology) or MATH 7830 Fiber Bundles (vector bundles, characteristic classes, elements of K-theory) and MATH 9800 Topology Seminar.
  In addition, during the second year students should take at least one additional topology course.

One of the courses MATH 7820, 7830 will be offered in the Spring semester each year; generally they will alternate. Besides taking the sequence MATH 5770, 7800, 7810, 7820/7830, students with interests in topology are required—within the first two years—to take at least one additional course in the specialized area or track of topology (see below) which they expect to follow. These additional courses may be independent reading courses taken under the supervision of a faculty member.

General Exam
The General Exam in Topology is based on the material of MATH 5770, 7800.

Research Seminar
Topology students are expected to take and to participate actively in MATH 9800 (Topology Seminar) every semester after the first year. Research seminars are an important component of the graduate program. Participation in them gives students an opportunity to be exposed to the current research in topology. In the *first semester of the second year*, students contemplating working in topology should contact a faculty member regarding a topic for a literature survey and, during the *second semester*, give a short expository talk in the Topology seminar.

Second-Year Proficiency Exam
In the second year, students take the Second-Year Proficiency Exam, which, in topology, consists of a conversation with a panel of faculty members on the material of two or three topology courses taken by the student during the second year, and on the bibliographical research done by the student and presented in MATH 9800.

3.2 Algebraic Topology

Faculty: G. Arone, M. Hill, and N. Kuhn

The subject of algebraic topology is the interplay between topology and algebra. One associates algebraic objects, e.g., groups and rings, with topological spaces in a ‘natural’ way, and investigates how the algebraic invariants reflect the topological structure of the spaces. Research in this area requires a good understanding of both topology and algebra. Areas of particular interest to faculty include homotopical algebra and homotopy as organized by the calculus of functors, group cohomology and its connections to representation theory and algebraic K-theory, and the study of complex oriented cohomology theories. There are deep connections with many parts of algebra, including algebraic geometry and number theory, and mathematical physics.

Recommended Advanced Courses
MATH 7840 Homotopy Theory, MATH 8800 Generalized Cohomology, MATH 7600 Homological Algebra, MATH 8700 Lie Groups, MATH 8650 Algebraic K-Theory, MATH 8830 Cobordism and K-Theory, and special topics (or reading courses) in calculus of functors and homotopical algebra.

3.3 Geometric Topology

Faculty: V. Krushkal and T. Mark

The central subject of geometric topology is the theory of manifolds, and in particular their classification. Research in higher-dimensional geometric topology includes the study of surgery and cobordism theory, characteristic classes, and group actions. Low-dimensional topology includes knot theory, quantum invariants of three-dimensional manifolds, geometric and differential 4-dimensional topology. This area has deep connections with algebraic topology, representation theory, geometric analysis and mathematical physics.

Recommended Advanced Courses
MATH 8750 Topology of Manifolds, MATH 8830 Cobordism and K-Theory, MATH 8720 Differential Geometry, MATH 7840 Homotopy Theory, and special topics courses (or reading courses) in characteristic classes, knot theory, 4-dimensional topology.

4 Graduate Program in the History of Mathematics

Faculty: Karen Parshall

The graduate program in the history of mathematics includes a component in the history of science taken within the Department of History. Students in the program must satisfy all of the requirements for the Ph.D. in Mathematics. In particular, they must complete the coursework in mathematics and perform satisfactorily on General Examinations in two areas before they are permitted to proceed toward the doctorate. Strong reading competency in either French or German is required for admission into the program, with strong reading competency required in the other language by the time dissertation research begins. Depending on a particular student's interests, other languages may also be required.

Program Coursework
The following is typical for a student in the graduate program in the history of mathematics:

- **First Year**
  First Semester: MATH 7310 (Real Analysis), MATH 7751 (Algebra I), MATH 5770 (General Topology), or one additional mathematics course (to be determined depending on the student's future historical interests).
  Second Semester: MATH 7320 (Real Analysis II), MATH 7752 (Algebra II), one additional mathematics course (again to be determined depending on the student's future historical interests). General exams are taken at the end of the summer after the first year.

- **Second Year**
  First Semester: MATH 7753 (Algebra III), HIEU 3321 (The Scientific Revolution) (taken as MATH 9999), MATH 7340 (Complex Analysis), or one additional mathematics course depending on specific interests and needs, MATH 7000, and MATH 9010.
  Second Semester: MATH 5010 (The History of the Calculus) or MATH 5030 (The History of Mathematics) (depending on the year), HIUS 3401 (The Development of American Science) (taken as MATH 9999), MATH 7800 (Algebraic Topology I), or one additional course depending on specific needs and interests. The summer after the second year involves directed readings geared toward the isolation of an eventual dissertation topic.

- **Third Year and Beyond**
  Additional mathematics courses to complete the number of hours required for the degree (chosen in consultation with the adviser), and any additional courses as needed (in, for example, language(s), history, or philosophy).

Research Seminar
Students are expected to participate actively in MATH 9010 History of Mathematics Seminar in all semesters.

Dissertation Proposal
For students in this program, the proposal defense replaces the Second-Year Proficiency Exam. It generally takes place before the end of the third year. Successful defense of the proposal represents "permission to proceed" to the dissertation phase of the program. The proposal is a written document (generally 30 to 40 pages in length, exclusive of bibliography) that is presented in a public forum. It

- details and justifies the historical questions the student proposes to explore in the dissertation;
- provides a detailed sketch of what the dissertation will cover, how it will be organized, and why;
- situates the proposed work within the broader literature of the history of science and mathematics; and
- provides a detailed (first approximation) of the dissertation's bibliography.

The Faculty and Research Interests

Listed below are the regular faculty of the Mathematics Department, along with selected recent publications.

In addition to the regular faculty, the department has a number of Whyburn Research Instructors and visiting faculty from other universities in residence in any given year. Colloquia and seminars

http://www.math.virginia.edu/seminar.htm

also bring in experts from other universities to lecture on important recent advances.

ABDELMALEK ABDESSELAM
Assistant Professor; Ph.D., *Ecole Polytechnique*, 1997. Mathematical physics, combinatorics and representation theory.

- A complete renormalization group trajectory between two fixed points, *Comm. Math. Phys.* 276(3) (2007), 727–772.
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PETER ABRAMENKO
Professor; Ph.D. and Habilitation, University of Frankfurt/Main, 1987 and 1995. Group theory and geometry.

- Twin Buildings and Applications to S-Arithmetic Groups, Lecture Notes in Mathematics 1641, Springer (1996).
- *Buildings: Theory and Applications*, Graduate Texts in Mathematics 248, Springer (2008) (with Ken Brown).

GREGORY ARONE
Professor; Ph.D., Brown University, 1993. Topology.

- Calculus of functors, operad formality, and rational homology of embedding spaces, *Acta Mathematica* 199(2) (2007), 153–198 (with P. Lambrechts and I. Volic).
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MIKHAIL ERSHOV
Assistant Professor; Ph.D., Yale University, 2005. Group theory.

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CHRISTIAN GROMOLL
Assistant Professor; Ph.D., University of California–San Diego.

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- Diffusion approximation of a processor sharing queue in heavy traffic, *Annals of Applied Probability* 14 (2004), 555–611.

ZORAN GRUJIĆ
Associate Professor; Ph.D., Indiana University, 1998. Nonlinear PDE's and fluid dynamics.

- Space analyticity for the Navier-Stokes and related equations with initial data in L^p, *J. Funct. Anal.* 152 (1998), 447–466 (with I. Kukavica).
- The geometric structure of the super-level sets and regularity for 3D Navier-Stokes equations, *Indiana Univ. Math. J.*, 50(3) (2001), 1309–1317.

IRA W. HERBST
Professor; Ph.D., University of California, Berkeley, 1971. Mathematical physics.

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MICHAEL A. HILL
Assistant Professor; Ph.D., Massachusetts Institute of Technology, Cambridge, 2006. Topology.

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JEFFREY J. HOLT
Associate Professor; Ph.D., University of Texas, 1993. Analytic number theory.

- The Beurling-Selberg extremal functions for a ball in Euclidean space, *Duke Math. J.* 83(1) (1996), 203–247 (with J. D. Vaaler).
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JOHN Z. IMBRIE
Professor; Ph.D., Harvard University, 1980. Physics.

- Branched polymers and dimensional reduction, *Ann. Math.* 158 (2003), 1019–1039 (with D. Brydges).
- End-to-end distance from the Green's Function for a hierarchical self-avoiding walk in four dimensions, *Commun. Math. Phys.* 239 (2003), 549–584 (with D. Brydges).

THOMAS L. KRIETE
Professor; Ph.D., University of Virginia, 1968. Operator theory.

- Toeplitz-composition C^*-algebras, *J. Operator Theory*, 359(6) (2007), 2915–2944 (with Barbara MacCluer and Jennifer Moorhouse).
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SLAVA KRUSHKAL
Professor; Ph.D., University of California-San Diego, 1996. Low-dimensional topology and geometry.

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NICHOLAS J. KUHN
Professor; Ph.D., University of Chicago, 1980. Algebraic topology and representation theory.

- Localization of Andre–Quillen–Goodwillie towers, and the periodic homology of infinite loopspaces, *Advances in Math.* 201 (2006), 318–378.
- Primitives and central detection numbers in group cohomology, *Advances in Math.* 216 (2007), 387–442.

IRENA LASIECKA
Professor; Ph.D., University of Warsaw, Poland, 1975. Nonlinear partial differential equations, mathematical control theory, applied mathematics.

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- Long time behavior of second order evolution equations with nonlinear damping (with I. Chueshov), *Memoires of AMS*, vol. 912, 2008.

BARBARA D. MacCLUER
Professor; Ph.D., Michigan State University, 1983. Function theory and operator theory.

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- Toeplitz-composition C^*-algebras, *J. Operator Theory*, 135–156 (with T. Kriete and J. Moorhouse).

THOMAS E. MARK
Assistant Professor; Ph.D., Michigan State University, 2000. Low-dimensional topology and geometry; gauge theory.

- Product formulae for Ozsvath-Szabo 4-manifold invariants, *Geom. Topol.* 12 (2008), 1557–1651 (with S. Jabuka).
- On the Heegaard Floer homology of a surface times a circle, *Adv. Math.* 218 (2008), 728–761 (with S. Jabuka).

KEVIN McCRIMMON
Professor; Ph.D., Yale University, 1965. Nonassociative algebras.

- *Taste of Jordan Algebras* (550 pages), Springer-Verlag Universitext, Berlin, 2004.
- The splittest Kac superalgebra K_{10}, *Journal of Algebra* 313 (2007), 554–589.

TAI MELCHER
Assistant Professor; Ph.D., University of California–San Diego, 2004. Stochastic analysis, geometry, PDEs.

- Hypoelliptic heat kernel inequalities on the Heisenberg group, *J. Functional Analysis* 221(2) (2005), 340-365 (with Bruce Driver).
- Malliavin calculus for Lie group-valued Wiener functions (submitted), arXiv:math.PR/0508419.

BRIAN PARSHALL
Gordon T. Whyburn Professor of Mathematics; Ph.D., Yale University, 1971. Representation theory.

- Reduced standard modules and cohomology, *Transactions of the American Mathematical Society*, to appear 2008 (with E. Cline and L. Scott).
- *Quantum Groups and Finite Dimensional Algebras*, Mathematical Surveys and Monographs, American Mathematical Society, vol. 150, 2008 (with B. Deng, J. Du, and J.-P. Wang).

KAREN V. H. PARSHALL
Professor; Ph.D., University of Chicago, 1982. History of mathematics.

- *James Joseph Sylvester: Jewish Mathematician in a Victorian World*, Baltimore: Johns Hopkins University Press, 2006.
- *Episodes in the History of Modern Algebra 1800–1950*, HMATH, vol. 32, Providence: American Mathematical Society and London: London Mathematical Society, 2008 (co-edited with Jeremy J. Gray).

DONALD E. RAMIREZ
Professor; Ph.D., Tulane University, 1966. Harmonic analysis and statistics.

- Anomalies in the Foundations of Ridge Regression, *International Statistical Review*, 76, 89–105 (2008) (with D. Jensen). http://arxiv.org/abs/math.ST/0703551
- Anomalies in the Analysis of Calibrated Data, *Journal of Statistical Computation and Simulation* 79(3) (2009), 299–314 (with D. Jensen).
  http://arxiv.org/abs/math.ST/0703550

ANDREI RAPINCHUK
Professor; Ph.D., Institute of Mathematics, Academy of Sciences of Belarus, 1982. Group theory and number theory.

- Valuation-like maps and the congruence subgroup property, *Invent. Math.* 144 (2001), 571–607 (with Y. Segev).
- Finite quotients of the multiplicative group of a finite dimensional division algebra are solvable, *J. Amer. Math. Soc.* 15 (2002), 929–978 (with Y. Segev and G. Seitz).

DAVID E. SHERMAN
Assistant Professor; Ph.D., University of California-Santa Barbara, 2001. Operator algebras.

- Noncommutative L^p structure encodes exactly Jordan structure, *Journal of Functional Analysis* 221 (2005), 150–166.
- Unitary orbits of normal operators in von Neumann algebras, *Journal für die Reine und Angewandte Mathematik* 605 (2007), 95–132.

LEONARD L. SCOTT
McConnell/Bernard Professor of Mathematics; Ph.D., Yale University, 1968. Group theory and representation theory.

- Semistandard filtrations in highest weight categories, *Michigan Math. J.* 58 (2009), 339–360.
- Reduced standard modules and cohomology, *Trans. Amer. Math. Soc.* 361 (2009), 5223–5261 (with Edward T. Cline and Brian J. Parshall).

LAWRENCE E. THOMAS
Professor; Ph.D., Yale University, 1970. Mathematical physics.

- Asymptotic behavior of thermal non-equilibrium steady states for a driven chain of anharmonic oscillators, *Comm. Math. Phys.* 215 (2000), 1–24 (with Luc Rey-Bellet).
- Exponential convergence to non-equilibrium stationary states in classical statistical mechanics, *Comm. Math. Phys.* 225 (2002), 305–329 (with Luc Rey-Bellet).

ROBERTO TRIGGIANI
Professor; Ph.D., University of Minnesota, Minneapolis, 1973. Control theory for partial differential equations, applied mathematics.

- *Control Theory for Partial Differential Equations: Continuous and Approximation Theories, Vols. 1 and 2* (with I. Lasiecka), Encyclopedia of Mathematics and its Applications, Cambridge University Press, 1,067 pp., January 2000. Volume 3 in preparation.
- Carleman estimates with no lower-order terms for general Riemannian wave equations. Global uniqueness and observability in one shot, *Applied Mathematics and Optimization, Special Issue in memory of J. L. Lions* (2002) 331–375 (with P. F. Yao).

WEIQIANG WANG
Professor; Ph.D., Massachusetts Institute of Technology, 1995. Representation theory.

- Representations of Lie superalgebras in prime characteristic I, *Proc. London Math. Soc.* (2009) (with Lei Zhao). arXiv:0808.0046.
- Modular representations and branching rules for wreath Hecke algebras, *IMRN* (2008), Article ID: rnn128–31 (with Jinkui Wan).
arXiv:0806.0196.

List by Research Area

Algebraic and Geometric Topology

Greg Arone, Mike Hill, Slava Krushkal, Nicholas Kuhn, Thomas Mark

Analytic Number Theory

Jeffrey Holt

Geometry

Peter Abramenko and Thomas Mark

Group Theory and Representation Theory

Peter Abramenko, Mikhail Ershov, Nicholas Kuhn, Brian Parshall, Andrei Rapinchuk, Leonard Scott, and Weiqiang Wang

Harmonic Analysis and Special Functions
Donald Ramirez

History of Mathematics
Karen Parshall

Mathematical Physics
Abdelmalek Abdesselam, Ira Herbst, John Imbrie, Lawrence Thomas, and Weiqiang Wang

Mathematical Statistics
Donald Ramirez

Nonassociative Algebra
Kevin McCrimmon

Operator Theory, Function Theory, and Operator Algebras
Thomas Kriete, Barbara MacCluer, and David Sherman

Partial Differential Equations, Applied Mathematics

Zoran Grujic, Irena Lasiecka, and Roberto Triggiani

Probability

Christian Gromoll and Tai Melcher

Real Analysis

Christian Gromoll and Tai Melcher

Courses

MATH 5010 - (3) (Y)
The History of the Calculus
Prerequisite: MATH 2310 and MATH 3351 or permission of instructor
Study of the evolution of the various mathematical ideas leading up to the development of the calculus in the seventeenth century, and how those ideas were perfected and extended by succeeding generations of mathematicians. Emphasizes primary source materials.

MATH 5030 - (3) (Y)
The History of Mathematics
Prerequisite: MATH 2310 and MATH 3351 or permission of instructor
Study of the development of mathematics from classical antiquity through the end of the nineteenth century, focusing on the critical periods in the evolution of such areas as geometry, number theory, algebra, probability, and set theory. Emphasizes primary source materials.

MATH 5100 - (3) (Y)
Mathematical Probability
Prerequisites: Three semesters of calculus, and graduate standing or departmental approval. Students who have received credit for MATH 3100 may not take MATH 5100 for credit
Study of the development and analysis of probability models through the basic concepts of sample spaces, random variables, probability distributions, expectations, and conditional probability. Additional topics include distributions of transformed variables, moment generating functions, and the central limit theorem.

MATH 5110 - (3) (Y)
Stochastic Processes
Prerequisite: MATH 3100 or permission of instructor
Topics in probability selected from: Random walks, Markov processes, Brownian motion, Poisson processes, branching processes, stationary time series, linear filtering and prediction, queuing processes, and renewal theory.

MATH 5140 - (3) (Y)
Mathematics of Derivative Securities
Prerequisite: MATH 2310 or MATH 1220 and a knowledge of probability and statistics. MATH 3100 or its equivalent is recommended.
Topics include arbitrage arguments, valuation of futures, forwards and swaps, hedging, option-pricing theory, and sensitivity analysis.

MATH 5210 - (3) (Y)
Advanced Calculus and Applied Mathematics
Prerequisites: MATH 2310, 3250; 3351 recommended.
Topics include vector analysis, Greens, Stokes, divergence theorems, conservation of energy, and potential energy functions. Emphasizes physical interpretation, Sturm-Liouville problems and Fourier series, special functions, orthogonal polynomials, and Greens functions.

MATH 5220 - (3) (Y)
Partial Differential Equations and Applied Mathematics
Prerequisites: MATH 5210
Introduces complex variables and partial differential equations. Topics include analytic functions, complex integration, power series, residues, conformal mapping; separation of variables, boundary value problems, Laplaces equation, wave equation, and heat equation.

MATH 5250 - (3) (IR)
Advanced Ordinary Differential Equations
Prerequisites: MATH 2310, 3250, 3351 or permission of instructor
Study of the qualitative geometrical theory of ordinary differential equations. Topics include all or most of the following: Picards method and basic existence and uniqueness theorems; linear systems; the phase plane and Sturms theorems; the Poincaré-Bendixon theorem; and Lyapunovs method and stability. Other topics presented as time permits.

MATH 526 - (3) (IR)
Partial Differential Equations
Prerequisite: MATH 2310, 3250 and 3351 or permission of instructor
A theoretical introduction from a classical viewpoint. Topics include harmonic and subharmonic functions; wave and heat equations; Cauchy-Kowalewski and Holmgren theorems; characteristics; and the Hamilton-Jacobi theory.

MATH 530 - (3) (IR)
Computer Methods in Numerical Analysis
Prerequisites: MATH 3351, 4300, and computer proficiency
A study of the underlying mathematical principles, and the use of sophisticated software for numerical problems such as spline interpolation, ordinary differential equations, nonlinear equations, optimization, and singular-value decomposition of a matrix.

MATH 5310 - (3) (Y)
Introduction to Real Analysis
Prerequisites: MATH 2310, 3351
Includes the basic topology of Euclidean spaces, continuity and differentiation of functions of a single variable, Riemann-Stieltjes integration, convergence of sequences and series.

MATH 5330 - (3) (Y)
Advanced Multivariate Calculus
Prerequisites: MATH 5310
Differential and integral calculus in Euclidean spaces, implicit and function theorems, differential forms, and Stokes Theorem.

MATH 5651 - (3) (Y)
Advanced Linear Algebra
Prerequisite: MATH 3351 or permission of instructor
This course includes a systematic review of the material usually considered in MATH 3351 such as matrices, determinants, systems of linear equations, vector spaces, and linear operators. However, these concepts will be developed over general fields and more theoretical aspects will be emphasized. The centerpiece of the course is the theory of canonical forms, including the Jordan canonical form and the rational canonical form. Another important topic is general bilinear forms on vector spaces. Time permitting, some applications of linear algebra in differential equations, probability, etc., are considered.

MATH 5652 - (3) (Y)
Introduction to Abstract Algebra
Prerequisite: MATH 3351 or permission of instructor
Focuses on structural properties of basic algebraic systems such as groups, rings and fields. A special emphasis is made on polynomials in one and several variables, including irreducible polynomials, unique factorization and symmetric polynomials. Time permitting, such topics as group representations or algebras over a field may be included.

MATH 5654 - (3) (Y)
Survey of Algebra
Prerequisite: MATH 1320 or equivalent and graduate standing
Surveys groups, rings, and fields, and presents applications to other areas of mathematics, such as geometry and number theory. Explores the rational, real, and complex number systems, and the algebra of polynomials.

MATH 5700 - (3) (Y)
Introduction to Geometry
Prerequisite: MATH 2310, 3351 or permission of instructor
Study of topics selected from analytic geometry, affine geometry, projective geometry, hyperbolic, and non-Euclidean geometry.

MATH 5720 - (3) (Y)
Introduction to Differential Geometry
Prerequisite: MATH 2310, 3351 or permission of instructor
Study of topics selected are from the theory of curves and surfaces in Euclidean space and the theory of manifolds.

MATH 5770 - (3) (Y)
General Topology
Prerequisite: MATH 3310 or the equivalent
Topological spaces and continuous functions, connectedness, compactness, countability and separation axioms, and function spaces. Time permitting, more advanced examples of topological spaces, such as projectives spaces, as well as an introduction to the fundamental group will be covered.

MATH 5830 - (3) (SI)
Seminar
Prerequisite: Permission of instructor
Presentation of selected topics in mathematics.

MATH 5896 - (3) (S)
Supervised Study in Mathematics
Prerequisite: Permission of instructor and graduate standing
In exceptional circumstances, a student may undertake a rigorous program of supervised study designed to expose the student to a particular area of mathematics. Regular homework assignments and scheduled examinations are required.

MATH 7250 - (3) (Y)
Ordinary Differential Equations and Dynamical Systems
Prerequisite: MATH 5310 and linear algebra, or the equivalent
Topics include well-posedness and stability of dynamical flows, attractors, invariant manifolds and their properties, dissipative and Hamiltonian systems.

MATH 7310 - (4) (Y)
Real Analysis and Linear Spaces I
Prerequisite: MATH 5310 or equivalent
Introduction to measure and integration theory.

MATH 7320 - (3) (Y)
Real Analysis and Linear Spaces II
Prerequisites: MATH 7310, MATH 7340 or equivalent
Study of additional topics in measure theory. Banach and Hilbert spaces, and Fourier analysis.

MATH 7340 - (4) (SI)
Complex Analysis I
Study of the fundamental theorems of analytic function theory.

MATH 7350 - (3) (Y)
Complex Analysis II
Prerequisite: MATH 7340 or equivalent
Study of the Riemann mapping theorem, meromorphic and entire functions, topics in analytic function theory.

MATH 7360, 7370 - (3) (Y)
Probability I and II
Prerequisite: MATH 7310 or equivalent
Rigorous introduction to probability, using techniques of measure theory. Includes limit theorems, martingales, and stochastic processes.

MATH 7410 - (3) (Y)
Functional Analysis I
Prerequisites: MATH 7340 and MATH 7310 or equivalent
Study of the basic principles of linear analysis, including spectral theory of compact and self adjoint operators.

MATH 7420 - (3) (SI)
Functional Analysis II
Prerequisite: MATH 7410 or equivalent
Study of the spectral theory of unbounded operators, semigroups, and distribution theory.

MATH 7450 - (3) (IR)
Introduction to Mathematical Physics
Prerequisite: MATH 5310
An introduction to classical mechanics, with topics in statistical and quantum mechanics, as time permits.

MATH 7751, 7752, (4) (Y)
Algebra I, II
Prerequisites: MATH 5651, 5652 or equivalent
Study of groups, rings, fields, modules, tensor products, and multilinear algebra.

MATH 7753 - (3) (Y)
Algebra III
Prerequisites: MATH 7751, 7752 or equivalent
Study of the Wedderburn theory, commutative algebra, topics in advanced algebra.

MATH 7754 - (3) (Y)
Algebra IV
Further topics in algebra.

MATH 7600 - (3) (SI)
Homological Algebra
Study of modules, algebras; Ext and Tor; cohomology of groups and algebras; differential graded modules, algebras, coalgebras; spectral sequences; and homological dimension.

MATH 7800 - (3) (Y)
Algebraic Topology I
Prerequisite: MATH 7751; corequisite: MATH 7752
The fundamental group and covering spaces. Simplicial and singular homology. Euler characteristic and degree. Classical applications including fixed point theorems.

MATH 7810 - (3) (Y)
Algebraic Topology II
Prerequisite: MATH 7800
The universal coefficient theorem, the Künneth formula. The cohomology ring: cup and cap products. Manifolds, orientations, the fundamental class, Poincaré duality.

MATH 7820 - (3) (Y)
Differential Topology
Prerequisite: MATH 7810 or permission of instructor
Smooth manifolds and functions, tangent bundles and vector fields. Embeddings, immersions, transversality. Regular values, critical points, degree of maps. Vector bundles, characteristic classes. Differential forms and de Rham cohomology.

MATH 7830 - (3) (Y)
Fiber Bundles
Prerequisite: MATH 7800
Topics include coordinate bundles; principal bundles and classifying spaces; vector bundles and characteristic classes; elementary K-theory.

MATH 7840 - (3) (Y)
Homotopy Theory
Prerequisite: MATH 7810
Topics include fibrations and cofibrations; homotopy groups; cohomology operations; Eilenberg-MacLane spaces; obstruction theory and spectral sequences.

MATH 8250 - (3) (SI)
Prerequisites: MATH 7410, MATH 7250
Partial Differential Equations
Study of topics in the theory of ordinary and partial differential equations.

MATH 8300 - (3) (SI)
Topics in Function Theory
Study of topics in real and complex function theory.

MATH 8310, 8320 - (3) (Y)
Operator Theory I, II
Study of topics in the theory of operators on a Hilbert space and related areas of function theory.

MATH 8360 - (3) (O)
Stochastic Calculus and Differential Equations
Prerequisite: MATH 7370
Study of the basic theory of stochastic differential equations and examples of its applications. Includes a review of the relevant stochastic process and martingale theory, stochastic calculus including Itos formula, existence and uniqueness for stochastic differential equations, strong Markov property, and applications.

MATH 8370 - (3) (O)
Topics in Probability
Prerequisite: MATH 7370
Selected topics in probability.

MATH 8400 - (3) (SI)
Harmonic Analysis
Study of Banach and C* algebras, topological vector spaces, locally compact groups, Fourier analysis. Topics selected by instructor.

MATH 8450 - (3) (Y)
Topics in Mathematical Physics
Application of functional analysis to physical problems; scattering theory, statistical mechanics, and quantum field theory.

MATH 8510 - (3) (SI)
Group Theory
Study of the basic structure theory of groups, especially finite groups.

MATH 8520 - (3) (SI)
Representation Theory
Study of the foundations of representation and character theory of finite groups.

MATH 8853 - (3) (SI)
Algebraic Combinatorics
Geometries, generating functions, partitions, and error-correcting codes and graphs are studied by using algebraic methods involving group theory, number theory, linear algebra and others.

MATH 8854 - (3) (SI)
Arithmetic Groups
Prerequisite: MATH 7752
General methods of analyzing groups viewed as discrete subgroups of real algebraic subgroups. Additional topics include the congruence subgroup problem.

MATH 8855 - (3) (SI)
Theory of Algebras
Study of the basic structure theory of associative or nonassociative algebras.

MATH 8600 - (3) (SI)
Commutative Algebra
Study of the foundations of commutative algebra, algebraic number theory, or algebraic geometry.

MATH 8620 - (3) (SI)
Algebraic Geometry
Study of the foundations of algebraic geometry.

MATH 8650 -(3) (SI)
Algebraic K-Theory
Topics include projective class groups and Whitehead groups; Milnors K2 and symbols; higher K-theory and finite fields.

MATH 8700 - (3) (Y)
Lie Groups
Study of basic results concerning Lie groups, Lie algebras, and the correspondence between them.

MATH 8710 - (3) (Y)
Lie Algebras
Study of basic structure theory of Lie algebras.

MATH 8720 - (3) (SI)
Differential Geometry
Study of differential geometry in the large; connections; Riemannian geometry; Gauss-Bonnet formula; differential forms, and other topics.

MATH 8750 - (3) (SI)
Topology of Manifolds
Study of manifolds from the topological, piecewise-linear, or smooth point of view; topics selected from imbeddings, smoothing theory, Morse theory, index theory, and s-cobordism.

MATH 8800 - (3) (SI)
Generalized Cohomology Theory
Topics include the axiomatic generalized cohomology theory; representability and spectra; spectra and ring spectra; orientability of bundles in generalized cohomology theory; Adams spectral sequence, and stable homotopy.

MATH 8830 - (3) (SI)
Cobordism and K-Theory
Study of classical cobordism theories; Pontryagin-Thom construction; bordism and cobordism of spaces; K-theory and Bott periodicity; formal groups, and cobordism.

MATH 8851 - (3) (Y)
Topics in Algebraic Topology
Study of selected advanced topics in algebraic topology.

MATH 8880 - (3) (SI)
Transformation Groups
Study of groups of transformations operating on a space; properties of fixed point sets, orbit spaces; and local and global invariants.

MATH 8995 - (3-12) (Y)
Thesis

MATH 8998 - (3-12) (Y)
Non-Topical Research, Preparation for Research
For masters research, taken before a thesis director has been selected.

MATH 8999 - (3-12) (Y)
Non-Topical Research
For masters thesis, taken under the supervision of a thesis director.

MATH 9010 - (3) (Y)
History of Mathematics Seminar

MATH 9250 - (3) (Y)
Differential Equations and Dynamical Systems Seminar

MATH 9310 (3) (Y)
Operator Theory Seminar

MATH 9360 - (3) (Y)
Probability Seminar

MATH 9410 - (3) (Y)
Analysis Seminar

MATH 9450 - (3) (Y)
Mathematical Physics Seminar

MATH 9950 - (3) (Y)
Algebra Seminar

MATH 9800 - (3) (Y)
Topology Seminar

MATH 9995 - (3-9) (Y)
Independent Research

MATH 9998 - (3-12) (Y)
Non-Topical Research, Preparation for Doctoral Research
For doctoral research, taken before a dissertation director has been selected.

MATH 9999 - (3-12) (Y)
Non-Topical Research
For doctoral dissertation, taken under the supervision of a dissertation director.

Admission Requirements

Thank you for your interest in mathematics graduate study at the University of Virginia. We hope you will decide to apply. If you do, you may obtain an application online:

http://artsandsciences.virginia.edu/admissions/apply.html

[We hope you do not find the University's online form too daunting. To view the application, you will have to create an account: this is easy to do. To browse the whole form before filling it out, go to a second page of the application, then click 'Postpone Data Validation.']

As the University's most international department, we welcome applications from **International Students**; such students should take particular note of the special instructions we list below.

Virginia residents should note the "Instate Educational Privilege Form" in the application package.

In making its decisions about admission and financial aid, the department is first and foremost looking for students that we feel will flourish in our program. To determine if a student will be successful, the department pays close attention to the overall undergraduate record, letters of recommendation, GRE scores (both on the general examinations and on the mathematics subject exam), and the student's personal statement.

Regarding the undergraduate record, the rigor of the applicant's mathematics program is an important factor; a good grounding in real analysis (as in our MATH 5310 and MATH 5330), algebra (as in MATH 5651/5652), and complex analysis (as in MATH 3340) is ideal. For students from smaller schools, we have found that a solid score on the Mathematics Subject Exam is a useful indicator of future success.

Students occasionally arrive with an area of interest in mind, planning to work with a specific research group: this is quite common for international students. Foreign applicants are particularly encouraged to contact our faculty about their specific mathematical interests.

Applicants whose native language is not English will need to demonstrate adequate English skills, but, to help with the application process of foreign students, we note that the mathematics faculty includes native speakers of Chinese, Russian, German, Italian, Polish, Croatian, and Hebrew.

Finally, we are looking for enthusiasm for mathematics in all its depth and beauty!

Admissions Checklist

To apply to our program, you will need to:

- fill out the basic application, which includes a personal statement,
- make arrangements to take the Graduate Record Exams (including the Mathematics subject test) and arrange to have the scores forwarded to us,
- have two or three letters of recommendation sent on your behalf, preferably from teachers who have taught you in your most substantial mathematics courses,
- arrange to have official transcripts mailed to the Graduate School, and
- submit the $60 application fee with your application to the Graduate School.

Additional Admissions Information for International Students

- Foreign applicants need to submit a TOEFL (or IELTS) score indicating English proficiency. This need not be submitted by international students who have already been enrolled for at least two years in an American university (e.g., in an undergraduate program). Taking the TOEFL internet-based test (iBT) is strongly encouraged because the speaking component is an important prerequisite in qualifying for teaching assistantships.

- On the online application form, foreign students without visas (e.g., those applying from overseas) should leave the questions about visa number and type blank.
- Once admitted, foreign students will need to submit the form 'Financial Guarantee and Personal Data for Foreign National Applicants.' This is needed by the University to process paperwork related to visas.

Two useful University of Virginia websites for international students are:

http://www.virginia.edu/iso/issp/student/admissions.htm

and

http://www.virginia.edu/intlstudents.html

To receive full consideration for admission and financial aid, your application should be complete by JANUARY 15.

History of Mathematics

Students interested in our program in the History of Mathematics must also provide evidence of reading proficiency in two foreign languages, generally French and German. In their personal statements, they should address not only their mathematical preparation but also their competence in history and their specific interests in the history of mathematics. They are encouraged to learn about our program by contacting Professor Karen Parshall well before the application deadline.

Transfer Students

With the approval of the Department and the Dean of the Graduate School, up to one session of the required three sessions of graduate work may be completed at another graduate school. A Master's degree from another school automatically counts as 24 hours toward the Ph.D. degree.

Contact Information

Questions about our program can be directed to the admissions officer,

Greg Arone (zga2m @ virginia.edu).

Graduate School of Arts and Sciences
P. O. Box 400775
University of Virginia
Charlottesville, Virginia 22904-4775
Telephone (434) 924-7184 (TDD: (434) 982-HEAR)

Admissions, Department of Mathematics
P. O. Box 400137
University of Virginia
Charlottesville, Virginia 22904-4137
e-mail: JulieR @ virginia.edu
Telephone (434) 924-4919 (TDD: (434) 982-HEAR)

Financial Support

There is no separate application form for financial aid; all applicants for admission are automatically considered.

Almost all mathematics doctoral graduate students receive full support throughout their study at the University, with financial aid awarded on an annual basis, contingent upon satisfactory progress. Financial aid for entering students typically consists of teaching fellowships, which involve assisting in discussion sections of undergraduate courses. After the first year, support typically consists of teaching assistantships, which involve teaching lower division undergraduate courses such as calculus. Typically, a small number of students are supported through competitive university fellowships, and the department's own Floyd Fellowship. Additional support in the summer has often been available to advanced students through research grants or summer session teaching.

The Department of Mathematics has received a U.S. Department of Education GAANN grant for graduate fellowships (GAANN standing for Graduate Assistance in Area of National Need). The grant will enable the Department to fund as GAANN fellows eight graduate students who are pursuing the Ph.D. degree. Five years of academic year and summer funding, including a dissertation-year fellowship, will enable successful fellows to maintain momentum towards their degrees. The average time-to-completion of the Department's previous GAANN fellows has been 4.8 years. The Department of Mathematics provides its graduate students with a broad exposure to mathematics and advanced training in mathematical research, as well as substantive teacher training. All of its graduate students may benefit from the summer program provided to the GAANN fellows, which include transitional classes during the summer prior to the official beginning of their program. The Department seeks students who show exceptional promise for research and teaching in mathematics, and who have demonstrated financial need. Individuals from backgrounds that have been traditionally under-represented in the mathematical sciences, are particularly encouraged to apply.

At the present time, academic year stipends range from $17,600 to $30,000, depending upon experience and duties, plus an allowance for health benefits.

Tuition and fees for 2009–2010 are $12,628 for Virginia residents and $22,685 for out-of-state students. Students receiving teaching assistantships typically have all tuition and fees waived.

International Students

On arrival, and before beginning as a teaching assistant, a student whose first language is not English will have their spoken English proficiency evaluated by the International Students Office and Teaching Resource Center. This test is similar to ETS's Test of Spoken English (TSE), and students who have taken the TSE, and have a high enough score, can be exempted from this evaluation.

Housing

Most students live off Grounds, where apartments cost an average of $450-750 per month. Apartments are available directly adjacent to the University area, as well as in the city of Charlottesville and Albemarle County. A free University Transit System provides access to many areas of off-Grounds housing.

Some University housing is available. Contact:

Director of Housing
P. O. Box 400735
Charlottesville, Virginia 22904-4735
Telephone: (434) 924-6873 (TDD: (434) 982-HEAR)
Fax: (434) 924-3758
Email: housing@virginia.edu

Housing information can also be found online:

http://www.virginia.edu/housing

Foreign students are encouraged to look at the University's webpage for international students:

http://www.virginia.edu/intlstudents.html

The University and Area

The University of Virginia was founded in 1819 by Thomas Jefferson, who established the curriculum and designed the buildings which still provide a focus for the University, allowing close contact between the students and faculty and promoting Jefferson's ideal of an "academical village" for the free exchange of ideas between fields. Today the University has grown to a community of 20,000 students, including 6,300 graduate and professional students, and nearly 2,000 faculty in ten schools. The faculty of Arts and Sciences, with undergraduate and graduate programs in 24 departments, defines, as it has from Jefferson's day, the focal point of the University.

The University is located in Charlottesville near the foothills of the Blue Ridge Mountains. The relatively mild but seasonal weather contributes to the area's reputation as an exceptionally pleasant place to live. The region is unmatched in the richness of historic sites, museums, and landmarks, and is known for its fine restaurants and local wineries. The University sponsors an extensive year-round program of events, including concerts, plays, and social and athletic activities. Charlottesville is 110 miles from Washington, D.C.; 70 miles from the state capital, Richmond; a 20-minute drive from the George Washington National Forest, and a few hours from the Chesapeake Bay, Atlantic Ocean, and Virginia's beaches. The airport offers direct service to Washington, D.C., Baltimore, Pittsburgh, Cincinnati, Charlotte, and New York.