Competition for attention in the information (overload) age

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Abstract

Limited consumer attention limits product market competition: prices are stochastically lower the more attention is paid. Ads compete to be the lowest price with other ads from the same sector and they compete for attention with ads from other sectors: equilibrium sector ad shares under free entry follow a CES form. When a sector gets more attractive, its advertising expands: others lose ad market share but may increase in absolute terms if sufficiently attractive. The "information hump" shows highest ad levels for intermediate attention levels when there is a decent enough chance of getting the message across and also of not being undercut by a cheaper offer. The Information Age takes off when the number of sectors grows, but total ad volume reaches an upper limit. Overall, advertising is excessive, though the allocation across sectors is optimal. Nonetheless, both large sectors and small ones can be blamed for misallocation of ads in using up scarce attention.

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1 Introduction

According to a Wiki cite, perhaps the first academic to articulate the concept of attention economics was Herbert Simon when he wrote

> ...in an information-rich world, the wealth of information means a dearth of something else: a scarcity of whatever it is that information consumes. What information consumes is rather obvious: it consumes the attention of its recipients. Hence a wealth of information creates a poverty of attention and a need to allocate that attention efficiently among the overabundance of information sources that might consume it. (Simon 1971, p. 40-41).

This is echoed in Lanham (2006), in the idea that we are drowning in information, but short of the attention to make sense of that information.[1] Our interest in this paper is in turning around Simon's point and looking how restricted attention affects the market information provided. In particular, we look at competition between firms providing information in the form of ads for their products. Facing limited attention, a firm might try and get away with a high price in the hope that its competitors' ads about their lower prices has been crowded out from the information receiver's attention span. This leads us to consider the dispersion of prices in the face of endogenous information congestion where information from each sector competes within the sector and with each other sector (even though sectors do not directly compete except for attention).

The Information Age is naturally captured in our framework as a result of several forces. One is a lower cost of sending information; more (and cheaper) channels now reach potential consumers. Traditional billboards and newspaper ads have been supplemented by Internet pop-ups, telemarketing, and product placements within TV programs (and on football players' jerseys). Information costs have not been lowered uniformly across the board, though, and some sectors' messages are more appropriately delivered by the new media. However, cheaper access to attention also means that rivals can access attention more cheaply too, intensifying in-sector competition. This effect renders competition more acute, lowering prices and benefiting

[1] See Eppler and Mengis (2004) for a multi-disciplinary review of Information Overload.

consumers. Scarcity of attention brings spillovers into other sectors, like raising their prices and making it more likely interesting offers are missed.

New products are also responsible for pricing churn and churn in the other advertising sectors too. A new sector tends to depress existing sectors' ads in relative terms (as a fraction of the total volume of messages), and it drives down weaker sectors in absolute terms. It may though cause stronger sectors to increase in size because price competition is relaxed (prices are stochastically higher). Thus there are information complementarities across product classes.

The third effect we track is the attention span of consumers. Since both work and leisure time are spent increasingly on information-carrying activities, it is likely that consumer attention spans have risen. This may induce more or less information transmission. When consumer attention is sparse, little information will be sent because there is not much chance of getting a look-in. Prices will be near monopoly levels because there is little chance of running across a rival. With a lot of attention, not much information is sent because there is a good chance the consumer will get a better offer from the same sector. Prices will be low, so the benefit from sending a message is low. The middle ground - the "information hump" - is the fertile ground for messages, yielding a fair shot at making a sale at a reasonably high price, both by being seen but no rival from the same sector being found.

We can also track the distribution of messages across sectors. With low levels of attention, highly profitable sectors will be most prominently represented. Increasing consumer attention brings firms into more competition with each other, which drives down sector profitability and serves to equalize opportunities across sectors while generally lowering mark-ups. Improved communication costs also lower prices, though improvements are sector specific, the extra crowding can relax competition (and raise prices) in other sector.

The framework we use to model firms' actions is adapted from Butters' (1977) seminal work on informative advertising, which is remarkable for delivering a tractable and intuitive description of equilibrium price dispersion. Butters derives a density of advertised prices and sales prices; he proposes a monopolistic competition framework distinct from that of Chamberlin (1933). In both the Butters and Chamberlinian formulations of monopolistic competition, the competitive part comes from a free-entry zero profit condition that closes the model. The monopolist part in Chamberlin's work comes from heterogeneity of the products sold by firms; in Butters it comes from the market power that firms have due to imperfect information that consumers do not know all firms' prices.

We meld Butters' approach with the advertising clutter approach formalized in Van Zandt (2005) and Anderson and de Palma (2007). Reception of messages is passive: the consumer does not search. This corresponds to getting messages from bulk mail, from the television, from billboards, etc. We focus on the interaction of multiple industries competing for individuals' attention. While Butters generates price dispersion because each individual gets only a subset of the price messages, we suppose that the individual misses some of the messages sent. This reflects advertising clutter because an individual is bombarded by too many messages (in "junk" mail, billboards, television, and internet pop-ups) to pay full attention to all.

Anderson and de Palma (2007) model both the consumer's choice of how much attention to supply and the actions of firms vying for that attention by sending messages advertising their wares.[2] The consumer's attention is a common property resource insofar as a message sender ignores the effects of its own message on other senders. This means there is a congestion externality, and a tax on messages can improve the allocation of resources.[3] However, one concern with this conclusion is that direct business-stealing effects are closed down in that model: message senders do not compete directly in the marketplace, they just compete for attention. A tax might *a priori* reduce price competition by reducing message volume, and so harm consumer welfare. We investigate this question investigated by specifically modeling competition within each of several sectors vying for consumer attention. The focus on firm competition necessitates simplifying the consumer side of the model: it is assumed here the consumer's attention span is fixed outside the model.

We first characterize an equilibrium model with interaction both within and across sectors. Competition within a sector means that a lower price is more likely to be the lowest sector offer in the set of messages the consumer has screened. Nonetheless, higher-price senders can remain in equilibrium: there is a trade-off between sales probability and mark-up, so all can earn zero profits despite price dispersion. Competition

[2] In a similar vein, Falkinger (2008) and Johnson (2008) also analyze consumer screening of message types (filtering).

[3] However, if the consumer's attention is not congested, a tax may worsen the allocation insofar as message senders do not internalize the consumer surplus from contacting prospective clients.

among sectors (industries) comes from overall competition for consumer attention, and price dispersion in each sectors depends on all other active sectors.

Surprisingly, the model endogenously generates an inverse IIA property for sector message fractions, and a CES form for the total number of messages sent. This bears an intriguing parallel to the CES utility functional form so often used to parameterize Chamberlinian models. Information congestion gives a new rationale for the CES specification, but it is now coupled with price dispersion within multiple sectors.

The model also generates a different welfare prescription from Butters (1977). While Butters' model has the optimal and equilibrium level of information equal, we find that the market allocation can be improved by taxing messages.[4] This reflects the property that advertising is excessive, in contrast to most of the theoretical economics literature on the subject (see Bagwell, 2007, for a survey). Indeed, the standard result in the economics of informative advertising is that there is not enough advertising because firms do not capture the consumer surplus. This is the monopoly result (see Shapiro, 1981, for example). Under oligopoly, this is somewhat offset by business stealing: overadvertising arises in the Grossman and Shapiro (1983) model of informative advertising when the business stealing effect outweighs the consumer surplus one.[5] Along similar lines, Stegeman (1991) shows that the market advertising is insufficient when the Butters model is amended to allow demand to have some elasticity: firms then tend to overprice without sufficient regard to the consumer surplus lost. In our context, over-advertising is quite natural as it serves to dissipate rents.

The next Section describes the model and solution technique. Section 3 derives the CES form for total advertising, derives the aggregates in the model and discusses their properties for information level changes. Section 4 finds the advertising and sales price distributions by sector, and ties them into the earlier comparative static results. Section 5 sets out the normative properties, the optimal allocation property and the tax prescription to deal with over-advertising. Section 6 describes extensions to allow for distractions and more general demand. Section 7 concludes. The Appendix gives a quick reminder of the Butters (1977) model.

[4] This finding reinforces the conclusion of Anderson and de Palma (2007), where the optimal policy in the presence of congestion was a tax on transmission.

[5] Excessive advertising is also found in the controversial Dixit and Norman (1978) paper on persuasive advertising.

2 Message reception and transmission

2.1 Assumptions

There are $\Theta > 1$ active commercial sectors indexed by $\theta = 1, ..., \Theta$.[6] Each active sector comprises a continuum of firms. These firms post messages and each message is an (ex-ante anonymous) advertisement containing the price at which a consumer can buy the product from the sending firm. Firms within each sector produce homogenous goods, and each sector transmits an endogenous number of messages, n_θ, for a total number of $N = \sum_{\theta=1}^{\Theta} n_\theta$ messages (per consumer). Each active firm sends just one message per consumer at a cost γ_θ (which can represent the cost of a letter, or the cost of a billboard divided by the number of consumers reached).[7] Hence n_θ also represents the number of firms in sector θ.

The cost of producing the good advertised in the message is c_θ (which is only incurred if the good is bought – think of a pizza for example): if the good must be produced beforehand regardless of whether the consumer buys, it suffices to set $c_\theta = 0$ and fold the production cost into the transmission cost, γ_θ.

Consumers are assumed to be identical. The cost of reaching any consumer is the same across consumers: messages could be sent to them by mail, or they could be posted on billboards, or on TV programs. However, *reaching* a consumer does not mean the message is *registered*. Each consumer has the same probability of registering a message (which means retaining the price offer). Since we assume constant returns to scale in production (constant marginal costs), we can treat the consumer as the unit of analysis and so we henceforth refer to a single consumer.[8]

The consumer registers a fixed number of messages, $\phi \geq 1$, which are drawn at random from the N messages sent. This reflects limited information processing capability. In what follows we will assume that $\phi < N$ in order to capture advertising clutter / information congestion. After registering the ϕ messages, the consumer makes her purchase decisions. She chooses the lowest priced offer received from each sector (we argue below that the probability of ties is zero) and buys q_θ units if that price is no larger than her reservation price for the sector, b_θ. We later allow the conditional demand to depend on price, but for the

[6] In Section 3.6 we discuss how these sectors are endogenously determined.

[7] Indeed, in equilibrium no sender would want to send a second message: to do so would give a negative profit given the original message just made a zero profit under the free-entry assumption below.

[8] Allowing for multiple consumer types would be useful for extending the model to analyze consumer targeting.

moment demand is rectangular.

Finally, we assume that the number of firms in each sector is determined by a zero-profit (free-entry) condition, as indeed is the density of messages in a sector at any advertised price.

2.2 Solution technique

A firm's expected demand is the probability that its message is registered and it is the lowest price received from its sector. Its expected demand also must satisfy the zero profit condition for the price charged. We equate the probability of making a sale at a particular price from these two different angles to find the relation between the price and the advertised price distribution.

The highest price set by any firm, b_θ, plays a key role because the only way a sender can make a profit at such a price is if it is the only message drawn from that sector. This ties down the number of messages n_θ sent from sector θ as a fraction of the total number of messages sent, N. Summing over sectors yields the total number sent, N, from which we can back out the number in each sector (the n_θ's). Armed with that statistic, we can recover the equilibrium price distribution in each sector and its support. This technique also enables us to determine endogenously the equilibrium number of active sectors.

More formally, an equilibrium to the model maps the primitives $\left\{\phi, \{\pi_\theta\}_{\theta=1,\ldots,\bar{\Theta}}\right\}$ into a set of non-negative sector message numbers $\{n_\theta\}_{\theta=1,\ldots,\bar{\Theta}}$, which define the total message volume as $N = \sum_{\theta=1}^{\bar{\Theta}} n_\theta$. A sector is active if and only if $n_\theta > 0$. For each active sector, the equilibrium specifies sector purchase probabilities, $\mathbb{P}_\theta$, for the consumer, and a price distribution within each sector, and corresponding choice probabilities for each product $\mathbb{P}(\theta, p)$. We show that equilibrium is unique, with an endogenous cut-off between active and inactive sectors. We proceed in Lemma 1 by determining message volume by sector as a function of the number, N, of total messages and active sectors (both variables to be determined later). We then sum over active sectors in Proposition 4, to find the N which must be consistent with the number of active sectors. Then, in Proposition 5 we find which are the active sectors. Intermediate results describe properties of the solutions.

In Section 3 we determine aggregate numbers of messages per sector and total messages, and in Section

4 we describe price dispersion within each sector.

2.3 Message selection probability

We first seek the probability of registering one particular message and registering no other message from the sector, θ, it came from. Assume $n_\theta < N$ (so at least two sectors are active). In the development in the main text we consider choice with replacement. This corresponds (loosely) to being exposed to a constant stream of messages, with repetition (e.g., billboards on a commute repeated daily). In a later footnote, we develop the appropriate expressions for choice without replacement; which might represent going through the day's bulk mail or email. Both formulations give the same choice probability, under the proviso that ϕ is small relative to N, which is the case we consider.

The probability that the first message drawn is the one under consideration is $1/N$. If draws are taken with replacement, the probability that none of the $n_\theta - 1$ other messages in the sector is registered on the subsequent $\phi - 1$ draws is $\left(1 - \frac{n_\theta - 1}{N}\right)^{\phi-1} \approx \left(1 - \frac{n_\theta}{N}\right)^{\phi-1}$.[9] The probability that the message under consideration is drawn is ϕ/N. If ϕ is small relative to N, then there is a negligible probability this same message is drawn twice (or more). Then, for $\phi << N$, the probability $\mathbb{P}_\theta$ that one (specific) message from sector θ is registered, and no other message is registered from that sector, is thus:[10]

$$\mathbb{P}_\theta = \frac{\phi}{N}\left(1 - \frac{n_\theta}{N}\right)^{\phi-1}. \tag{1}$$

This is conveniently rewritten as $\mathbb{P}_\theta = \frac{\phi}{N - n_\theta}(1 - \mathbb{Q}_\theta)$,[11] where

$$\mathbb{Q}_\theta = 1 - \left(1 - \frac{n_\theta}{N}\right)^{\phi}. \tag{2}$$

Here $\mathbb{Q}_\theta$ is readily seen as the probability that there is *at least one hit* in the sector (θ): the probability

[9] Without replacement, the probability that the first of the $\phi - 1$ other draws is not from sector θ is $\left(1 - \frac{n_\theta - 1}{N-1}\right) \approx \left(1 - \frac{n_\theta}{N}\right)$. By extension, the probability that *none* of the other $\phi - 1$ messages gets through, assuming $\phi << N$, is

$$\left(1 - \frac{n_\theta - 1}{N-1}\right)\left(1 - \frac{n_\theta - 1}{N-2}\right)...\left(1 - \frac{n_\theta - 1}{N - (\phi - 1)}\right) \approx \left(1 - \frac{n_\theta}{N}\right)^{\phi-1}.$$

With is a large number of messages, drawing one message does not noticably change the residual number of messages in the sector. On the other hand, because ϕ is an integer, one draw does significantly reduce the number of other draws left.

[10] We will later use the notation $\mathbb{P}(p, \theta)$ to denote the probability of a sale at price p in sector θ: hence $\mathbb{P}_\theta = \mathbb{P}(b_\theta, \theta)$, since we shall show that a sale at the top price sent, b_θ, only happens when the message is the only one drawn from the sector.

[11] This is the probability that the sector is not chosen (second term) times the probability that the message is chosen given ϕ draws *outside* the sector (first term). This makes sense once one realizes that the individual message in question, being "small", can just as well be effectively housed initially outside the sector.

that each of the n_θ messages is missed on each of the ϕ draws. The second important link between the two probabilities is that $\frac{\partial \mathbb{Q}_\theta}{\partial n_\theta} = \mathbb{P}_\theta$: the increased chance of discovering a sector when an extra message is sent is the probability that the extra message is registered when no other message from the sector has registered.

3 Advertising levels

3.1 Advertising shares by sector

Consider an advertisement which is sent out for a price equal to the reservation price b_θ. As we argue in Section 4 below, there will be such an ad, and the probability of finding a second ad at the same price is zero. Since $\mathbb{P}_\theta$ (as given by (1)) is the probability this is the only ad found from sector θ, the equilibrium zero profit condition reads:

$$(b_\theta - c_\theta)\, q_\theta \mathbb{P}_\theta = \gamma_\theta, \tag{3}$$

where we recall that q_θ is the quantity of good θ demanded. Define π_θ by:

$$\pi_\theta = \frac{(b_\theta - c_\theta)\, q_\theta}{\gamma_\theta} > 1,$$

which measures the economic potential (surplus per \$ transmission cost) of sector θ. The zero profit condition (3) for the equilibrium probability the highest-priced sender in active sector θ makes a sale is then

$$\mathbb{P}_\theta = \frac{1}{\pi_\theta}. \tag{4}$$

This probability depends only on the intrinsic economic performance index, π_θ, of the sector.

Let $\bar{\Theta}$ be the number of sectors for which $\pi_\theta > 1$: this is the *maximum* number of active sectors. We rank these sectors such that π_θ is decreasing in the index θ, i.e. from highest to lowest economic performance. For simplicity (except when we do the symmetric analysis) we will assume that all the π_θ's are different across sectors. In the sequel, we will find the endogenous number of active sectors, which we denote $\Theta \leq \bar{\Theta}$. It is necessary (but not sufficient) for an active sector that $\pi_\theta > 1$ because $(b_\theta - c_\theta)\, q_\theta$ must exceed γ_θ in order for the sender to wish to incur the cost of a message, given that messages are not read with certainty.

Lemma 1 *Let $N > \phi$. All θ such that $\pi_\theta > \frac{N}{\phi}$ are active sectors, and the rest are inactive. The relative sector sizes are*

$$\frac{n_\theta}{N} = \max\left\{0, 1 - \left(\frac{N}{\phi}\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}\right\}, \qquad \theta = 1, ..., \bar{\Theta}. \tag{5}$$

Proof. Equating the probability derived from the zero-profit condition, (4), with the probability as derived from the individual's sampling that she gets no other message from the subset in the sector, (1), implies $\mathbb{P}_\theta = \frac{\phi}{N}\left(1 - \frac{n_\theta}{N}\right)^{\phi-1} = \frac{1}{\pi_\theta}$, and so determines the ad market shares by rewriting this as (5). Hence, sector θ sends a positive number of messages if and only if $\pi_\theta > \frac{N}{\phi}$. ■

If $\pi_\theta \leq \frac{N}{\phi}$, then even a single message sent from the sector at the highest price would not be expected to cover its costs: i.e.,

$$(b_\theta - c_\theta)\, q_\theta \frac{\phi}{N} \leq \gamma_\theta, \tag{6}$$

where $\frac{\phi}{N}$ is the probability the message is registered.[12] We defer considering the overall comparative static properties of equilibrium because N is still to be determined in (5). However, we can use the expression to compare across sectors of different economic characteristics within an equilibrium.

Sectors with larger economic potential send more messages because they are more attractive to senders. That is, $n_\theta > n_{\theta'}$ if and only if $\pi_\theta > \pi_{\theta'}$. We proceed by further characterizing the relation that sector sizes must satisfy at any equilibrium.

3.2 The inverse IIA property

Sector message sizes exhibit a type of IIA property (Independence of Irrelevant Alternatives) in the sense that the ratio of ad market shares of two sectors depends only on their profitabilities *for a given N.* However, contrary to the usual IIA property (first pointed out by Debreu (1960) in his critique of Luce's (1959) Choice Axiom), which stipulates that the ratio of market shares does not change with the number and type of other options, this ratio *does* change here since N changes with the profitability of a third sector (see also (9) below). Thus, the standard IIA property does *not* hold for this model.

[12] As we shall see below, this is also the condition for the lowest price in the price support to be below b_θ. (For the lowest price, γ_θ equals the mark-up times the probability of being drawn. The latter is ϕ/N since a sale is guaranteed for the lowest price in the sector, conditional on being drawn. Since the low price is critically at b_θ, the condition follows immediately.)

However, a related IIA property holds, with respect to the market shares of all competing sectors. We call this the *inverse* IIA property, which pertains to the ratios $m_{-\theta} \equiv \frac{N-n_\theta}{N}$, where n_θ is the number of messages from sector θ. From (5), the inverse IIA property is:[13]

$$\frac{m_{-\theta}}{m_{-\theta'}} = \left(\frac{\pi_{\theta'}}{\pi_\theta}\right)^{\frac{1}{\phi-1}}. \tag{7}$$

This is a property of invariance of the ratio of *all rivals'* advertising levels as the appeal of any rival (outside the pair) changes. Analogously to the way the IIA property implies the Logit model, the inverse IIA property implies an inverse Logit formulation:[14]

Proposition 2 *At any equilibrium with Θ active sectors, the non-θ shares have a logit form:*

$$\frac{m_{-\theta}}{(\Theta-1)} = \frac{\pi_\theta^{\frac{-1}{\phi-1}}}{\sum_{\theta'=1}^{\Theta} \pi_{\theta'}^{\frac{-1}{\phi-1}}} \equiv \Psi_\theta, \quad \theta = 1, ..., \Theta, \tag{8}$$

where the LHS is the non-share of sector θ over the total non-share of all sectors.

Proof. Inverting (7),

$$\frac{m_{-\theta'}}{m_{-\theta}} = \left(\frac{\pi_\theta}{\pi_{\theta'}}\right)^{\frac{1}{\phi-1}}.$$

Summing over θ' gives $\frac{(\Theta-1)}{m_{-\theta}} = (\pi_\theta)^{\frac{1}{\phi-1}} \sum_{\theta'=1}^{\Theta} \pi_{\theta'}^{-\frac{1}{\phi-1}}$, and the result follows directly by inversion. ■

Some care must be taken with the interpretation of the result. In particular, the value of Θ is endogenous here (and is determined below), and so only the active sectors are counted: inactive sectors π_θ must be excluded from the summation.[15] The same caveat applies below.

As π_θ increases, the RHS of (8) falls: the more attractive is a sector, then the more its ads push out the ad shares of other sectors. That is, as profitability rises, the affected sector produces proportionately more ads while the others produce relatively less.[16] Even a mature sector may enjoy a higher profitability if γ_θ falls, perhaps because of the advent of a new medium which might complement advertising its goods. The model says that sectors which benefit from such improved communication costs get larger ad market shares

[13] It would be interesting to check empirically whether this property is satisfied when the standard IIA property is violated (see Train, 2003, for a discussion of the Hausman test for IIA in the context of discrete choice models).

[14] Therefore $\frac{n_\theta}{n_{\theta'}} = \frac{1-(\Theta-1)\Psi_\theta}{1-(\Theta-1)\Psi_{\theta'}}$, which indicates that IIA does not hold.

[15] This is true too in the standard logit insofar as only available options are included when determining choice probabilities.

[16] We see in Section 3.7 that the number of ads from sector θ' may actually rise if that sector is sufficiently attractive.

at the expense of the others. Indeed, as shown in sections 3.6 and 3.7, weak sectors might be pushed out of the market entirely.

The effects of raising ϕ on the distribution of messages by sector are fundamentally those of the logit formulation (see for example Anderson, de Palma, and Thisse (1992)), though the derivation of that form above differs from the usual roots.

Proposition 3 *For Θ constant, as ϕ rises, the ad market share of the most profitable sector decreases with ϕ, and the share of the least profitable sector increases. As ϕ falls to 1, almost all messages are sent by the most profitable sector.*

Proof. To show the first point, fix Θ. The relation in (8) gives the fraction of messages in sector θ as $\frac{n_\theta}{N} = 1 - (\Theta - 1)\Psi_\theta$. Note that $\frac{d\pi_\theta^{\frac{-1}{\phi-1}}}{d\phi} = -\frac{\pi_\theta^{\frac{-1}{\phi-1}}}{(\phi-1)^2}\ln\frac{1}{\pi_\theta}$, so that

$$\frac{d\Psi_\theta}{d\phi} \overset{s}{=} -\ln\frac{1}{\pi_\theta} + \sum_{\theta'=1}^{\Theta}\Psi_{\theta'}\ln\frac{1}{\pi_{\theta'}},$$

or (since $\sum_{\theta'=1}^{\Theta}\Psi_{\theta'} = 1$),

$$\frac{d\Psi_\theta}{d\phi} \overset{s}{=} \sum_{\theta'=1}^{\Theta}\Psi_{\theta'}\left(\ln\frac{1}{\pi_{\theta'}} - \ln\frac{1}{\pi_\theta}\right).$$

Hence, the share decreases with ϕ for the most profitable sector (1), and increases for the least profitable one (Θ).

We now show that the limit case $\phi \downarrow 1$ involves a single viable sector. First note from (8) that

$$\Psi_1 = \frac{\pi_1^{\frac{-1}{\phi-1}}}{\sum_{\theta=1}^{\Theta}\pi_\theta^{\frac{-1}{\phi-1}}} = \frac{1}{1+\sum_{\theta=2}^{\Theta}\left(\frac{\pi_1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}}.$$

Hence, $\lim_{\phi\downarrow 1}\Psi_1 = 0$: almost all messages are sent from sector 1. ■

If the attention span is very limited (ϕ close to 1), virtually all messages are from the highest profit sector, 1, because this yields the greatest profit conditional on making "the" hit. The messages sent tend to quote the monopoly price because there is almost no chance of being undercut by another message. Monopoly prices are most attractive for the sector with the highest monopoly profit. The number of messages sent from this sector tends to π_1.[17] This corresponds to pure dissipation of the monopoly profit in sector 1. It is

[17] This can be seen as follows. If N messages are sent, all from sector 1, and one is drawn, then monopoly pricing implies the profit from a message is $\frac{b_1-c_1}{N}q_1 - \gamma_1$. The zero profit condition implies the number of messages is π_1.

possible that there is a huge number of such messages if π_1 is very high: even if π_2 is high too (but strictly below π_1), it attracts virtually no messages. This case arises if the transmission cost for one sector tends to zero while the other sectors retain positive costs: the sector crowds out all other sectors. This is clearly wasteful because all other sectors are closed out, while the affected sector just dissipates all the rents in excessive message transmission.[18]

At the other extreme, when the attention span is extensive, any price above the lowest in the sector will almost certainly be beaten. All sectors are very competitive, so sectors become equally (un)attractive: a lot of price competition means very few messages per sector.

When $\Theta > 2$, the advertising shares of the intermediate sectors are not necessarily monotonic in the level of consumer attention, ϕ. To see this, consider 3 sectors. Sectors 1 and 2 have very high profits, with 2 slightly less than 1, while sector 3 has very low profit. When the attention span is slightly above one message, sector 1 is active while 2 is virtually silent. For middling values of ϕ, both 1 and 2 have almost half the market each. For ϕ large, all have around one third shares. Sector 2's share is not monotonic here.

Expression (8) in turn gives rise to a familiar functional form.

3.3 Aggregate advertising

The next step is to determine the equilibrium message volume, N. Expressions (5) and (8) give two different expressions for m_{-i}. Equating them yields:[19]

Proposition 4 *The equilibrium total message size given $\Theta > 1$ active sectors takes a CES form:*

$$N = \phi (\Theta - 1)^{\phi-1} \Big/ \left(\sum_{\theta=1}^{\Theta} \pi_\theta^{\frac{-1}{\phi-1}}\right)^{(\phi-1)} . \quad (9)$$

Thus N is increasing in each profitability, π_θ, and homogenous of degree one in the sector profitabilities.

The CES form has well-known properties.[20] The first property means that raising the profitability of any sector causes the total volume of messages to rise: the extra clamor causes a larger total without a fully

[18] As we shall see below, if all sector transmission costs fall proportionately to zero, the range of prices stays the same in each sector: the density of messages sent at any price simple rises proportionately (to the cost decrease) for all sectors.
[19] N can also be derived from summing up the expressions for market shares in (5).
[20] For example, it is maximal at symmetry (under the constraint that the sum of the inverse π_θ's is constant).

compensating backlash from the other sectors. Similarly, adding another viable sector raises N. To see the second point, consider introducing a "barely viable" sector s with $n_s = 0$: by (5), the corresponding attractivity of such a new sector s is $\pi_s = N/\phi$. We now verify that introducing this barely viable sector s leaves (9) unchanged:

$$\frac{N}{\phi} = \frac{(\Theta - 1)^{\phi-1}}{\left(\sum_{\theta'=1}^{\Theta} \pi_{\theta'}^{\frac{-1}{\phi-1}}\right)^{\phi-1}} = \frac{(\Theta)^{\phi-1}}{\left(\sum_{\theta'=1}^{\Theta} \pi_{\theta'}^{\frac{-1}{\phi-1}} + \left(\frac{N}{\phi}\right)^{\frac{-1}{\phi-1}}\right)^{\phi-1}}.$$

Thence, introducing a strictly viable sector, with $\pi_s > N/\phi$, will cause N to increase.[21]

The homogeneity property in Proposition 4 implies that total message volume doubles if all communication costs are halved.[22] This is one obvious cause of a surfeit of information: spam email is an everyday manifestation of the problem. Any such cost improvement is offset by the rise in messages sent, so all improvements are completely dissipated.[23]

We next consider the symmetric case before going into more detail into the asymmetric one.

3.4 The Information Age

One driver of the information age is lower communication costs, another is a larger set of viable sectors. Under symmetry ($\pi_\theta = \pi$ for all $\theta = 1...., \bar{\Theta}$), the expression (from (9)) for the total number of messages, N, reduces to[24]

$$N = \phi \left(\frac{\bar{\Theta} - 1}{\bar{\Theta}}\right)^{\phi-1} \pi. \tag{10}$$

Having more sectors, $\bar{\Theta}$, raises the total number of messages. The number N is a logistic function of the number of sectors: it is first convex (for $\bar{\Theta} < \phi/2$), and then concave, for $\bar{\Theta} > \phi/2$. If we were to view the number of (new) sectors as arriving at a constant rate, then this means the amount of information would accelerate at first (the take-off of the Information Age) before tapering off, reminiscent of the Bass (1969)

[21] That is, $\frac{N}{\phi} = \left[\Theta / \sum_{\theta'=1}^{\Theta} \pi_{\theta'}^{\frac{-1}{\phi-1}} + \left(\frac{N}{\phi}\right)^{\frac{-1}{\phi-1}}\right]^{\phi-1} < \left[\Theta / \sum_{\theta'=1}^{\Theta} \pi_{\theta'}^{\frac{-1}{\phi-1}} + \pi_s^{\frac{-1}{\phi-1}}\right]^{\phi-1}$..

[22] No further sectors will enter, since doubling of the existing message volume will preclude them, even if their transmission costs halve. Indeed, as we just noted, a sector is viable if and only if $\pi_s > N/\phi$.

[23] This is reminiscent of Zahavi's Law in transportation, which says that average travel times have remained constant over several decades, despite substantial increases in travel speed.

[24] Symmetric CES models are commonly deployed in the economics of product differentiation. Note here that the sector viability constraint, $\pi > N/\phi$, is automatically satisfied.

diffusion of innovation model. Indeed, the amount of information has an asymptote of $\bar{N} = \phi\pi$, which is the bound to the amount of information the system can sustain.[25]

The average number of messages per sector, $n_\theta = N/\bar{\Theta}$, is increasing in $\bar{\Theta}$ if and only if $\phi > \bar{\Theta}$, so it is eventually decreasing (for $\bar{\Theta}$ large enough). The interesting feature here is the initial increase. This is explained by the idea that more sectors mean less competition, so higher prices and more incentive to send messages.

The logistic function in (10) is sketched in Figure 1, for $\pi = 20$ and $\phi = 20$ (the asymptote of the function is at $N = 400$, the maximal value of $N/\bar{\Theta}$ is attained at $\bar{\Theta} = 20$, and the inflection point is at $\bar{\Theta} = 10$).

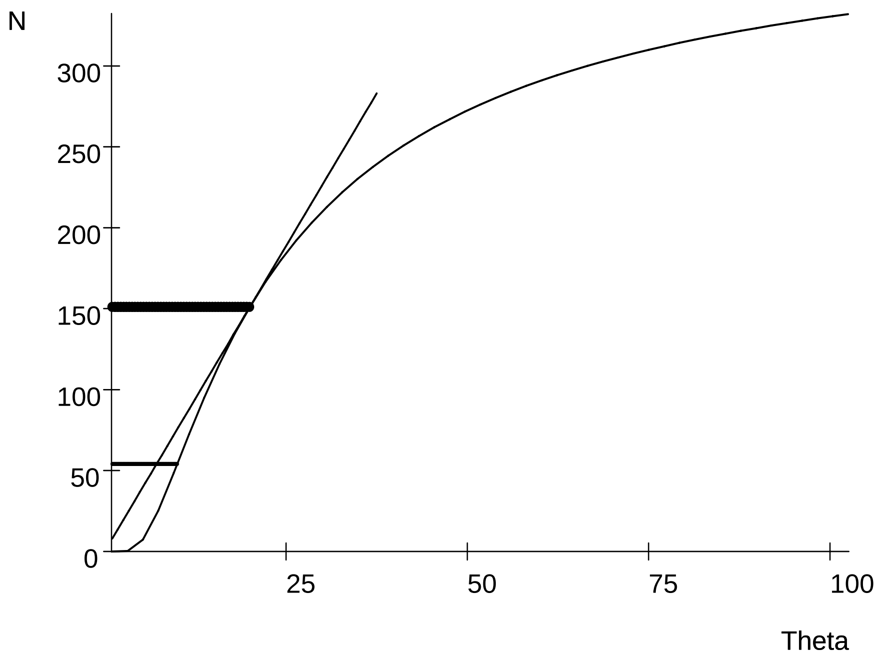

Figure 1. Total messages as a function of number of sectors.

The other comparative static property of N, with respect to ϕ, is described next.

[25] At the limit, monopoly prices, b, are set in each sector, returning π when the message is chosen. The probability of being chosen is $\phi/\bar{N}$, which therefore equals $1/\pi$ (see also (4)).

3.5 The information overflow hump

The advent of new media means consumer time is now spent with additional ad-carrying activities, like surfing the internet or sending email. This likely implies an increase in the consumer attention span as new ways arise to communicate. The thumbnail capture in the model of this increased span is to raise ϕ.

From the symmetric analysis (see (10)), we can derive how the information level, N, varies with the attention span, ϕ. Indeed, N is decreasing in ϕ if and only if $\phi > \hat{\phi} \equiv \frac{1}{\left(\ln \frac{\bar{\Theta}}{(\bar{\Theta}-1)}\right)}$, and so N is necessarily decreasing for $\phi > \bar{\Theta}$ (since $\bar{\Theta} \ln \frac{\bar{\Theta}}{(\bar{\Theta}-1)} > 1$: the LHS is decreasing in $\bar{\Theta}$ and goes to 1 as $\bar{\Theta}$ goes to infinity). Likewise, $\frac{N}{\phi}$ is falling in ϕ, and therefore N increases more slowly than ϕ.

Figure 2 plots the relation of N as a function of ϕ for $\pi = 100$ and $\bar{\Theta} = 10$ (hence $N = 100\phi \left(\frac{9}{10}\right)^{\phi-1}$ attains its maximum at $\hat{\phi} = \frac{1}{\left(\ln \frac{10}{9}\right)}$, which is slightly less than 10). The dashed line is the line $\phi = N$.

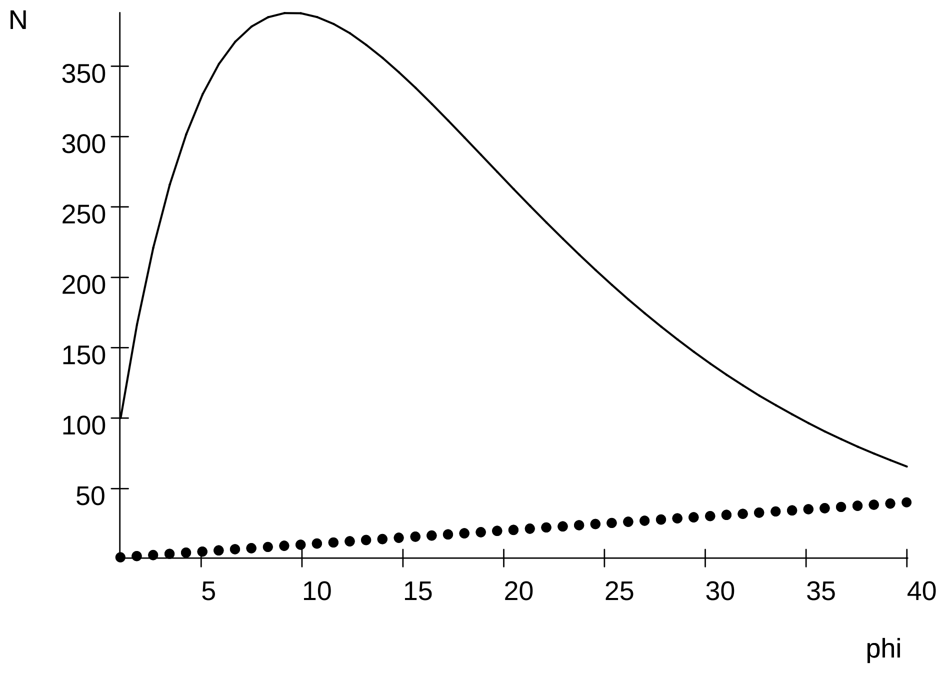

Figure 2. Total number of messages sent, N, as a function of examination, $\phi \geq 1$.

Figure 2 shows the quasi-concave function, i.e., first increasing, then decreasing with the attention span,

ϕ. This we term the *information overflow hump.* However, the number of messages only increases for low $\phi < \hat{\phi}$ ($< \bar{\Theta}$). More attention has two conflicting consequences. First, it raises the probability a message from the sector is seen, which raises profitability, and hence the number of messages sent, ceteris paribus. But it also has the effect of increasing price competition (the price distribution shifts down), as it is more likely a lower price will be found in the sector. This reduces profitability and leads to a smaller number of firms (messages). For low ϕ, the price competition effect is weak in that it is quite unlikely that another message received will be from the same sector as one already received: extra messages will most likely come from unrepresented sectors. With high reception rates, the price effect dominates. In a nutshell, for low ϕ and given $\bar{\Theta}$, more examination leads to more messages sent as undiscovered sectors become more likely to be found. For higher ϕ, more examination means more hits in the same sector, which increases price competition and so decreases sector activity.

3.6 Sector viability

When sectors are asymmetric, some may be precluded by the strength of those in the market. We now make precise the conditions for sectors to be active.

Let $\bar{\Theta}$ denote the number of sectors for which $\pi_\theta > 1$, and assume that $\bar{\Theta} > 1$.[26] Any sector with $\pi_\theta \leq 1$ is not viable, and so can be eliminated from the discussion (even if a message sent at the sector monopoly price were examined exclusively with probability one, it would not generate a profitable sale).

Proposition 5 *Assume that $\bar{\Theta} > 1$. Then there exists a unique equilibrium where all sectors $1, ..., \Theta$ are active, with $\Theta \in [2, \bar{\Theta}]$, and the total volume of messages is given by (9).*

Proof. From Lemma 1, a sector is active in equilibrium if $\pi_\theta > \frac{N_\Theta}{\phi}$, where we (temporarily) let N_Θ denote the number of messages for Θ active sectors as given by (9). We first show that if there are two sectors, then they are both active. From (9), $\frac{N_2}{\phi} = \frac{1}{\left(\pi_1^{\frac{-1}{\phi-1}} + \pi_2^{\frac{-1}{\phi-1}}\right)^{\phi-1}}$, and the RHS is less than π_2 (as is readily seen by cross-multiplying), so this implies $\pi_2 > \frac{N_2}{\phi}$, as desired for the second sector to be active.

[26] The model is degenerate if there is a single sector. From (9), there are zero messages.

Next, we show there is a unique sector cut-off, Θ. The condition for a sector to be active is $\pi_\theta > \frac{N_\Theta}{\phi}$. Given the ranking of sectors, the LHS decreases in the marginal sector, Θ, while we showed in the argument following Proposition 4 that the RHS increases as sectors are added. Note that all $\bar{\Theta}$ sectors are active if $\pi_{\bar{\Theta}} > \frac{N_{\bar{\Theta}}}{\phi}$ (which condition we showed to hold in the symmetric equilibria analyzed previously).

Finally, it remains to show that the equilibrium does indeed follow the ranking: that is, there cannot be an equilibrium with some sector θ excluded while some sector $\theta' > \theta$ is included. Suppose there were: then the profit from sending a single message from sector θ (at its monopoly price, b_θ) is $\pi_\theta \frac{\phi}{N}$. However, messages sent from sector θ' return a profit of at most $\pi_{\theta'} \frac{\phi}{N}$. Hence, since $\pi_\theta > \pi_{\theta'}$, a message from sector θ would supplant one from sector θ', so the starting point cannot be an equilibrium.[27] ■

Viability constraints imply that equilibrium congestion across sectors may close down a sector when another sector becomes more attractive. Similarly, a newly entering sector raises the congestion on the incumbents. These we illustrate next.

3.7 Raising a sector's profitability

We noted in Section 3.2 that an increase in a sector's profitability will increase the total number of messages sent. Since the other sectors all send smaller shares of this larger total, the affected sector must send more messages. We now determine what happens to the other sectors. Recall $\frac{n_\theta}{N} = 1 - \left(\frac{N}{\phi}\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}$ from (5). Hence for an unaffected sector (where π_θ has not changed) it is clear that the sector *share* goes down. However, it is possible the *number* of messages it transmits goes up, as we now show (that is, we show that $\frac{dn_\theta}{d\pi_{\theta'}}$ can be positive). Indeed, $\frac{dn_\theta}{d\pi_{\theta'}} = \frac{dn_\theta}{dN}\frac{dN}{d\pi_{\theta'}}$ has the sign of $\frac{dn_\theta}{dN}$ since $\frac{dN}{d\pi_{\theta'}} > 0$. From (5), we have the derivative[28]

$$\begin{aligned}\frac{dn_\theta}{dN} &= 1 - \frac{\phi}{\phi-1}\left(\frac{N}{\phi}\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}} \\ &= 1 - \frac{\phi}{\phi-1}\frac{(\Theta-1)}{\Theta}\frac{\pi_\theta^{\frac{-1}{\phi-1}}}{\left(\sum_{\theta'=1}^{\Theta}\pi_{\theta'}^{\frac{-1}{\phi-1}}\right)/\Theta},\end{aligned}$$

[27] If there are several sectors with the same profitability, then they are either all active or all inactive.

[28] From which we see that higher $\pi_{\theta'}$ increases the likelihood that the expression is positive.

where we have substituted $\frac{N}{\phi}$ from (9). Define $\chi_\theta = \pi_\theta^{\frac{-1}{\phi-1}}$ and so

$$\frac{dn_\theta}{dN} = 1 - \frac{\phi}{\phi-1}\frac{(\Theta-1)}{\Theta}\frac{\chi_\theta}{\bar{\chi}}, \tag{11}$$

where the average value of χ_θ, denoted by $\bar{\chi}$, is homogenous of degree $\left(\frac{-1}{\phi-1}\right)$ in the π_θ.

From a symmetric starting point (where $\chi_\theta = \bar{\chi}$ for all θ), $\frac{dn_\theta}{dN}$ has the sign of $1 - \frac{\phi}{\phi-1}\frac{(\Theta-1)}{\Theta}$, which is negative if and only if $\phi > \Theta$. If though $\phi < \Theta$, a marginally higher attractivity in one sector causes message numbers to *rise* in all sectors.

This result is broadly consistent with the rising part of the information hump (low ϕ) and for the early "take-off" part of the Information Age evolution depicted in Figure 1 (low Θ). In all cases, there is a relatively large increase in the number of messages sent as long as the amount of competition is small.

In the asymmetric case, (11) indicates that there is a cut-off value of χ_θ for which $\frac{dn_\theta}{dN}$ is negative for higher χ_θ and positive for lower χ_θ. Since π_θ is inversely related to χ_θ, this means that *larger sectors are more likely to see an increase* in the number of messages sent. A summary Proposition:

Proposition 6 *The equilibrium total message volume increases as any sector becomes more profitable. The improved sector sends more messages both relatively and absolutely. All other sectors diminish in relative importance, but sufficiently profitable sectors may increase the absolute number of messages sent.*

We now turn to the price distribution, whose properties underpin the economics of the results so far.

4 Equilibrium price dispersion

The equilibrium *sales probability* corresponding to a particular price p in sector θ can be determined independently of the other sectors (although the aggregate message volume, N, and attention span, ϕ, both matter). However, we need to bring in the other sectors to determine which prices are actually used in equilibrium. The equilibrium sales probability for a message announcing price p in sector θ, $\mathbb{P}(p,\theta)$, is given simply from the zero-profit condition as

$$\mathbb{P}(p,\theta) = \frac{\gamma_\theta}{(p-c_\theta)\,q_\theta} = \frac{(b_\theta - c_\theta)}{(p-c_\theta)}\frac{1}{\pi_\theta}, \tag{12}$$

where $\mathbb{P}(p,\theta) \in (0,1)$ for all p in the interior of the support of the equilibrium price distribution. The above expression reduces to the zero-profit condition (4), when $p = b_\theta$, and using the notation $\mathbb{P}(b_\theta,\theta) = \mathbb{P}_\theta$.

The equilibrium sales probability above is decreasing and convex in p. We next want to use it to determine the equilibrium *advertised price* distribution. We first argue that the support of the equilibrium advertised price distribution (for any active sector) is a compact interval $[\underline{p_\theta}, b_\theta]$ with no atoms nor gaps, where the lower bound, $\underline{p_\theta}$, is to be determined below. There are *no atoms* in the price distribution because if there were, any sender choosing the same price as a mass of other senders would raise profits by infinitesimally cutting its price. This would leave its mark-up essentially unchanged but raise sales discretely because it then beats all others at the purported mass point whenever two lowest price messages were the same. The interval has *no gaps* on the support because if there were, the lower price at a gap can be raised leaving the sales probability unchanged but increasing the mark-up. This same argument implies the support must go up to b_θ: if it stopped short, the highest price firm could raise its price with no penalty on sales probability and increase its mark-up. Finally, the lower bound of the support must exceed $c_\theta + \gamma_\theta/q_\theta$ because at any lower price the transmission cost cannot be recuperated.

Lemma 7 *Prices in industry θ are distributed on a compact support $[\underline{p_\theta}, b_\theta]$ where $\underline{p_\theta} > c_\theta + \gamma_\theta/q_\theta$, and there are no atoms.*

We now derive the lowest price in the support along with the equilibrium advertised price distribution. Let $F(p,\theta)$ denote the *fraction* of messages in sector θ sent at price p or below. (Then $F(\underline{p_\theta},\theta) = 0$ and $F(b_\theta,\theta) = 1$). A message at price p is successful as long as the price is the lowest one received: using the same logic as used to derive (1), the sales probability is

$$\mathbb{P}(p,\theta) = \frac{\phi}{N}\left(1 - \frac{n_\theta F(p,\theta)}{N}\right)^{\phi-1},$$

where we simply note that the number of messages sent from the sector with a price no higher than p is $n_\theta F(p,\theta)$.[29] Since $\mathbb{P}(p,\theta)$ is given by the zero profit condition (12), we have

$$\frac{(b_\theta - c_\theta)}{(p - c_\theta)}\frac{1}{\pi_\theta} = \frac{\phi}{N}\left(1 - \frac{n_\theta F(p,\theta)}{N}\right)^{\phi-1}, \tag{13}$$

[29] Clearly, expected sales per consumer of the cheapest priced product are $q_\theta\phi/N$ for sector θ.

where n_θ/N is given by (5).

Proposition 8 *The equilibrium advertised price density in sector θ is decreasing and convex on $\left[\underline{p_\theta}, b_\theta\right]$, with (truncated) Pareto distribution*

$$F(p,\theta) = \frac{1-\left(\frac{N}{\phi\pi_\theta}\right)^{\frac{1}{\phi-1}}\left(\frac{b_\theta-c_\theta}{p-c_\theta}\right)^{\frac{1}{\phi-1}}}{1-\left(\frac{N}{\phi\pi_\theta}\right)^{\frac{1}{\phi-1}}}, \tag{14}$$

where N is given by (9) and $\underline{p_\theta}$ is given by

$$\underline{p_\theta} = c_\theta + \left(\frac{N}{\phi}\right)\frac{\gamma_\theta}{q_\theta}. \tag{15}$$

Proof. The equilibrium advertised price distribution is given from the relation (13) as

$$F(p,\theta) = \frac{N}{n_\theta}\left(1-\left(\frac{N}{\phi\pi_\theta}\frac{b_\theta-c_\theta}{p-c_\theta}\right)^{\frac{1}{\phi-1}}\right).$$

Recalling that $\frac{n_\theta}{N} = 1-\left(\frac{N}{\phi\pi_\theta}\right)^{\frac{1}{\phi-1}}$ from (5), we can write (14). It is readily checked that $F(b_\theta,\theta)=1$.

Since $F\left(\underline{p_\theta},\theta\right)=0$, the lowest price in sector θ is determined by (13) as:

$$\left(\underline{p_\theta}-c_\theta\right) = \frac{(b_\theta-c_\theta)}{\pi_\theta}\frac{N}{\phi}.$$

Then (15) follows immediately. The corresponding density, $f(p,\theta)$ is strictly positive on $\left[\underline{p_\theta}, b_\theta\right]$, where it is decreasing and convex (as shown by differentiation of (14)). ■

The intuition for the lowest price in the support is straightforward. A message sent at this lowest price always beats all the other messages from the sector. Hence the sales probability is just the probability that it is read at all, which is simply $\frac{\phi}{N}$ since it has ϕ shots from a pool of N messages. Equating this probability times the mark-up to the cost of sending the message gives (15).

As in Butters (1977), lower prices are advertised more heavily. In the Butters model, the corresponding lowest price, $\underline{p}$, would be simply $c_\theta + \gamma_\theta/q_\theta$,[30] because such a price just covers the cost of production plus sending the message. In the Butters version, the lowest price must always get a sale because there is no information congestion, and no possibility that the message remains unread. In contrast, here the lowest

[30] This (trivially) allows for a quantity effect, which Butters does not have.

price in any sector does not always make a sale. Here, information overflow pushes up the lowest price in the support: this is needed to compensate for the likelihood that the message may not be received.

The simplest measure of price dispersion is the breadth of the support of the equilibrium prices. This is $b_\theta - \underline{p_\theta}$, where $\underline{p_\theta} = c_\theta + \left(\frac{N}{\phi}\right)\frac{\gamma_\theta}{q_\theta}$. Ceteris paribus, dispersion is smaller the greater is $\left(\frac{N}{\phi}\right)\frac{\gamma_\theta}{q_\theta}$ (recall though that N depends on all the parameters of the model, apart from the inactive sectors' profitabilities). Hence, for example, a larger γ_θ decreases N and so *increases* dispersion in all *unaffected* sectors, while decreasing dispersion in the affected sector (see (9)).

Changes within the sector affect the support as well as the aggregate message volume N. A sector can become inviable if it faces tough competition from other sectors and/or it is quite unattractive itself. Viability can be expressed as the condition that the price support does not collapse. That is $\underline{p_\theta} < b_\theta$. Writing out the condition, it means that $\frac{N}{\phi} < \pi_\theta$; this is the same condition from (5) for $n_\theta > 0$ in equilibrium.

The next two sub-sections stress the properties of the equilibrium price distribution with respect to two key variables of emphasis in the paper, sector profitability and consumer attention span.

4.1 Advertised price dispersion and sector profitability

Greater sector profitability impacts the affected sector by increasing the volume of messages sent (Proposition 4). As we now see (Proposition 9), this increases price competition, and so stochastically lowers prices. However, this market mechanism spills over into the other sectors. Elsewhere, price competition is reduced because sector messages are crowded out. Nonetheless, the number of messages sent in other sectors can actually rise (see Proposition 6) because the reduced price competition can raise profits per firm (which then must be reduced by further entry).

Proposition 9 *An increase in the attractivity of one sector decreases prices (and increases the support of price dispersion) in that sector and increases prices (and decreases the support of price dispersion) in the other sectors, in the sense of First-Order Stochastic Dominance. A proportional increase in the attractivity of all sectors leaves the price distribution unchanged.*

Proof. Recall $F(p,\theta) = \frac{1-\left(\frac{N}{\phi\pi_\theta}\right)^{\frac{1}{\phi-1}}\left(\frac{b_\theta - c_\theta}{p-c_\theta}\right)^{\frac{1}{\phi-1}}}{1-\left(\frac{N}{\phi\pi_\theta}\right)^{\frac{1}{\phi-1}}}$ by (14). Hence $\frac{dF}{d\pi_{\theta'}}$ (for $\theta' \neq \theta$) has the opposite sign from $\frac{dN}{d\pi_{\theta'}}$, which is positive, as already established. Hence $F(p,\theta)$ decreases in $\pi_{\theta'}$. However, $\frac{dF}{d\pi_\theta}$ has the opposite sign, since $\frac{N}{\pi_\theta}$ is decreasing in π_θ (from (9)). Hence, $F(p,\theta)$ increases in π_θ. If π_θ increases, $\underline{p_\theta}$ falls; if $\pi_{\theta'}$ increases, N rises so that $\underline{p_\theta}$ rises (see (15)).

If all sectors increase proportionately in attractivity, $\frac{N}{\pi_\theta}$ is unchanged (by the homogeneity in Proposition 4) and so $F(p,\theta)$ is unchanged. ■

This means that advertised prices (and price dispersion) can be negatively correlated across sectors. If one sector becomes more desirable (in the sense of higher surplus), prices fall in that sector as competition intensifies. But the additional messages crowd out messages in other sectors, and this relaxes competition in those other sectors. On the other hand, across-the-board changes affecting all sectors can leave prices the same. This property underlies the result in the next Section that proportionately lower message transmission cost savings are dissipated fully: equivalently, a (proportional) tax might be raised without deadweight loss.

The *sales price distribution* differs from the advertised price distribution because lower prices are more likely to get sales, and also because even the lowest advertised price does not always make a sale. It is derived in the Appendix. In the meantime, we follow through with the analysis of the symmetric case.

4.2 Dispersion and symmetric sectors

In the symmetric case, N is given by (10) as $N = \phi\left(\frac{\bar{\Theta}-1}{\bar{\Theta}}\right)^{\phi-1}\pi$, and so the cumulative distribution function for advertised prices (14) becomes

$$F(p,\theta) = \bar{\Theta}\left(1 - \frac{\bar{\Theta}-1}{\bar{\Theta}}\left(\frac{b-c}{p-c}\right)^{\frac{1}{\phi-1}}\right), \quad \text{for } p \in [\underline{p}, b], \tag{16}$$

where $\underline{p} = c + \left(\frac{\bar{\Theta}-1}{\bar{\Theta}}\right)^{\phi-1}(b-c)$ (by (15)).[31] Hence, as ϕ rises, the lower bound $\underline{p}$ falls, and so intra-sector competition rises in this respect. A tighter characterization is quite immediate.

Proposition 10 *Assume sectors are symmetric. A higher examination rate, ϕ, lowers prices in the sense of First-Order Stochastic Dominance.*

[31] As $\left(\frac{\Theta-1}{\Theta}\right)^{\phi-1}\tilde{\pi} \to 1$, then $N \to \phi$, and, (by (15)), $\underline{p}$ goes to $c + \frac{\gamma}{q}$: this is like the Butters price.

Proof. From (16), $F(p,\theta,\phi_2) > F(p,\theta,\phi_1)$ as $\left(\frac{b-c}{p-c}\right)^{\frac{1}{\phi_2-1}} < \left(\frac{b-c}{p-c}\right)^{\frac{1}{\phi_1-1}}$, or $\phi_1 < \phi_2$. ■

Lower prices as attention goes up underpins the earlier comparative static results of the information hump. Even though the total message volume is not monotone (see Figure 2), the price effect is. For low ϕ, prices are high and few messages are sent: for high ϕ, prices are low and few messages are sent. In the first case, because few messages are registered, firms may as well set high prices and chance the low probability of another message from the same sector. In the second case, price competition intensifies because there is a strong likelihood another message from the same sector will be read.

Along similar lines, it is readily shown that higher $\bar{\Theta}$ stochastically increases prices (with more price dispersion). This is because the limited attention is more divided.

We now turn to the normative analysis.

5 Normative properties

One strong property of the Butters (1977) model is that the market allocation is optimal. However, this property crucially depends on his assumption that each message hits somewhere.[32] In our set-up, there is rent dissipation and socially wasteful duplication of messages.[33] Competition for attention imposes a congestion externality which leads to excessive advertising: this feature is perhaps more in tune (rather than optimality or under-advertising) with one's personal reaction to advertising clutter.

The welfare function is given by summing over sectors the total sector surplus times the probability a sale is made in the sector, and then subtracting the message costs. Recall that $\mathbb{Q}_\theta$ is the probability of at least one hit in sector θ (see (2)), and write this as $\mathbb{Q}(n_\theta, N) = 1 - \left(1 - \frac{n_\theta}{N}\right)^\phi$, which depends only on own message fraction and the attention span. Then we can write the welfare function (for any values $n_\theta \geq 0$, $\theta = 1, ..., \bar{\Theta}$) as

$$W(n_1, ..., n_{\bar{\Theta}}; N) = \sum_{\theta=1}^{\bar{\Theta}} \left[(b_\theta - c_\theta) q_\theta \mathbb{Q}(n_\theta, N) - \gamma_\theta n_\theta\right], \tag{17}$$

[32] It also depends on the rectangular demand assumption. Stegeman (1991) shows that there is insufficient advertising if demand slopes down, because the pricing distortion has firms not internalizing the consumer surplus of lower prices. We discuss downward sloping demand below.

[33] Clearly the first best optimum comprises one message per sector, and the active sectors should be the ϕ for which the profit per message, $(b_\theta - c_\theta) q_\theta - \gamma_\theta$, is highest. If γ is the same for all θ, these are the first ϕ ones, meaning the ones for which π_θ is highest.

where $N = \sum_{\theta=1}^{\bar{\Theta}} n_\theta$. This form (breaking out N as a separate argument) is convenient for what follows.

Lemma 11 *The social benefit from an extra message in sector θ is equal to*

$$\frac{dW}{dn_\theta} = \frac{\partial W}{\partial n_\theta} + \frac{\partial W}{\partial N} = \left(\left(b_\theta - c_\theta\right) q_\theta \mathbb{P}_\theta - \gamma_\theta\right) + \frac{\partial W}{\partial N}, \tag{18}$$

where the RHS terms are private sector profit and congestion externality respectively.

Proof. From (17), we have $\frac{dW}{dn_\theta} = \frac{\partial W}{\partial n_\theta} + \frac{\partial W}{\partial N}\frac{dN}{dn_\theta}$: noting that $\frac{dN}{dn_\theta} = 1$ (message anonymity) gives (18).

Now, from (17) and (2), and then using (1), we have that

$$\begin{aligned} \frac{\partial W}{\partial n_\theta} &= \left(b_\theta - c_\theta\right) q_\theta \frac{\partial \mathbb{Q}\left(n_\theta, N\right)}{\partial n_\theta} - \gamma_\theta \\ &= \left(b_\theta - c_\theta\right) q_\theta \mathbb{P}_\theta - \gamma_\theta. \end{aligned} \tag{19}$$

This expression is the profit of a firm setting the top price in sector θ given n_θ messages emanating from the sector (see (4)). Since this is zero in equilibrium, the remaining term, $\partial W/\partial N$, is naturally interpreted as the congestion externality. ∎

The next result shows the externality is negative, and quantifies it at the equilibrium allocation.

Proposition 12 *The total number of messages transmitted is excessive in equilibrium, and the (negative) congestion externality is measured as the average transmission cost.*

Proof. Recall that $\frac{dW}{dn_\theta} = \frac{\partial W}{\partial n_\theta} + \frac{\partial W}{\partial N}$, and $\frac{\partial W(n^e,N^e)}{\partial n_\theta} = 0$ (where the superscript e denotes that the variable is evaluated at its equilibrium value) by the zero profit condition of equilibrium for all active θ. Then we have that $\frac{dW(n^e,N^e)}{dn_\theta} = \frac{\partial W(n^e,N^e)}{\partial N}$. From (17), we have

$$\begin{aligned} \frac{\partial W\left(n, N\right)}{\partial N} &= \sum_{\theta=1}^{\Theta} \left(b_\theta - c_\theta\right) q_\theta \frac{d\mathbb{Q}\left(n_\theta, N\right)}{dN} \\ &= -\sum_{\theta=1}^{\Theta} \left(b_\theta - c_\theta\right) q_\theta \frac{n_\theta}{N} \mathbb{P}_\theta. \end{aligned} \tag{20}$$

Using the zero profit condition (3) we get

$$\frac{\partial W\left(n^e, N^e\right)}{\partial N} = -\frac{1}{N^e}\sum_{\theta=1}^{\Theta} \gamma_\theta n_\theta^e < 0, \tag{21}$$

i.e., the congestion externality is strictly negative and equal to (minus) the average transmission cost. ■

This result underscores the main problem with the market equilibrium: although (as we show next) the allocation is optimal across sectors given the total equilibrium message volume, the *overall volume is excessive*. This is seen clearly from what we just argued in Lemma 11, namely that $\frac{\partial W(n^e,N^e)}{\partial n_\theta} = 0$ (i.e., evaluated at the equilibrium N), while $\frac{dW(n^e,N^e)}{dN} < 0$. However, while optimal and private incentives are aligned in terms of allocation, the private choice ignores the message crowding externality on all other sectors, which is measured by $\frac{\partial W(n^e,N^e)}{\partial N} < 0$. This implies excessive messages are sent. The social cost of an extra message, as per (21), is the average sending cost. This relation holds because if extra messages have to be sent, they should be allocated across sectors in proportion to the sector representation in the population: one more message therefore costs the average transmission cost.

Proposition 13 *The equilibrium allocation of messages across sectors is socially optimal given the number of messages transmitted at the equilibrium.*

Proof. Let N be given at the equilibrium level stipulated by (9), that we denote as N^e, and we wish to show that the *division* of these messages effectuated in equilibrium is optimal.

First, note that maximization of $W(.)$ under the constraint that the non-negative n_θ's sum to a given value of N is a maximization problem of a continuous function on a compact set and therefore must have a solution. Therefore at least one of the n_θ must be positive: call this sector j.

Second, substituting the constraint $n_j = N - \sum_{\theta\neq j}^{\bar\Theta} n_\theta$ into $W(n_1, ., n_j, ., n_{\bar\Theta}; N)$ enables us to write the maximand as $\tilde W(n_1, ., [n_j], .., n_{\bar\Theta}; N)$, and we now show that $\tilde W(.)$ is concave in the arguments $n_1, ., [n_j], .., n_{\bar\Theta}$ (for given N), where the notation $[n_j]$ denotes that the corresponding argument, n_j, is excluded. Indeed,

$$\begin{aligned}\tilde W(n_1, ., [n_j], .., n_{\bar\Theta}; N) &= (b_j - c_j)\, q_j \mathbb{Q}\left(N - \sum_{\theta\neq j}^{\bar\Theta} n_\theta, N\right) - \gamma_j\left(N - \sum_{\theta\neq j}^{\bar\Theta} n_\theta\right)\\ &\quad + \sum_{\theta\neq j}^{\bar\Theta}\left[(b_\theta - c_\theta)\, q_\theta \mathbb{Q}(n_\theta, N) - \gamma_\theta n_\theta\right].\end{aligned}$$

Recall that the sum of concave functions is concave. The terms in the transmission costs γ are linear in $n_1, ., [n_j], .., n_{\bar\Theta}$, while for $\theta \neq j$, the $\mathbb{Q}(n_\theta, N)$ terms are concave in own n_θ. There remains the term

$\mathbb{Q}\left(N - \sum_{\theta \neq j}^{\bar{\Theta}} n_\theta, N\right) = 1 - \left(\frac{\sum_{\theta \neq j}^{\bar{\Theta}} n_\theta}{N}\right)^\phi$ (by definition (2)): the summation term is linear in the n_θ, given N; hence raising this to a power $\phi > 1$ gives a convex function, and one minus a convex function is concave, as desired.

Third, since $\tilde{W}(.)$ is concave, and is maximized over a compact and convex set, it has a unique maximal value. Let a solution be denoted $\left\{n_1^o, .., \left[n_j^o\right], .., n_{\bar{\Theta}}^o\right\} \geq 0$, with $n_j^o = N^e - \sum_{\theta \neq j}^{\bar{\Theta}} n_\theta^o$, and let $\left\{\mu_1^o, .., \left[\mu_j^o\right], .., \mu_{\bar{\Theta}}^o\right\} \geq 0$ be the corresponding Lagrangian multipliers. The solution maximizes $\tilde{W}$ if and only if $\left\{n_1^o, ., \left[n_j^o\right], .., n_{\bar{\Theta}}^o; \mu_1^o, ., \left[\mu_j^o\right], .., \mu_{\bar{\Theta}}^o\right\}$ solves the Karush-Kuhn-Tucker conditions. This means that:

$$\frac{\partial \tilde{W}}{\partial n_\theta} = \begin{cases} = 0 & \text{if} \quad n_\theta^o > 0, \\ \leq 0 & \text{if} \quad n_\theta^o = 0. \end{cases}$$

By (19) we have $\frac{\partial \tilde{W}}{\partial n_\theta} = \left(\left(b_\theta - c_\theta\right) q_\theta \mathbb{P}_\theta - \gamma_\theta\right) - \left(\left(b_j - c_j\right) q_j \mathbb{P}_j - \gamma_j\right)$.

Therefore

$$\left(\left(b_\theta - c_\theta\right) q_\theta \mathbb{P}_\theta - \gamma_\theta\right) \quad \begin{cases} = \left(\left(b_j - c_j\right) q_j \mathbb{P}_j - \gamma_j\right) & \text{if} \quad n_\theta^o > 0, \\ \leq \left(\left(b_j - c_j\right) q_j \mathbb{P}_j - \gamma_j\right) & \text{if} \quad n_\theta^o = 0. \end{cases} \tag{22}$$

By the zero profit condition for active firms (3), $\left(b_\theta - c_\theta\right) q_\theta \mathbb{P}_\theta = \gamma_\theta$ if $n_\theta > 0$; but $\left(b_\theta - c_\theta\right) q_\theta \mathbb{P}_\theta \leq \gamma_\theta$ for inactive sectors (see (6)). This means that the market allocation solves (22), and so induces the maximal $\tilde{W}(.)$ and hence the maximal $W(.)$ under the constraint. In other words, as per (19), $\frac{\partial W}{\partial n_\theta} = 0$ by the zero profit condition for the highest-priced sender in sector θ, and so the equalization condition is guaranteed at the equilibrium N^e. ■

The key feature here is the one that generates the optimality result. This is that the marginal change in the choice probability holding fixed the total number of messages, $\frac{\partial \mathbb{Q}(n_\theta, N)}{\partial n_\theta}$, which is instrumental in the social problem, is equal to $\mathbb{P}_\theta$, the probability the highest-priced firm makes a sale in the private problem. The equivalence holds because the probability that an extra message is examined and nothing else was examined from the sector both reflects its social contribution and the private incentive for sending it. In neither case are we concerned about it crowding out other messages from the sector: in the private case, any other message takes precedence by dint of its lower price, and, in the social case, again only the extra likelihood of being examined counts.

5.1 Increasing transmission costs

In the next sub-section we look at taxes, but before doing so, we derive some stronger results that even cost increases without any corresponding revenue collection can improve the allocation. These results stress the extent of the market failure, and also help indicate which sectors are particularly responsible.

Proposition 14 *A uniform percentage increase in transmission costs leaves welfare unchanged. Price dispersion remains unchanged, as does the fraction of messages sent per sector, while the number of messages per sector (and therefore the total) goes down in proportion to the percentage cost increase. The number of active sectors remains the same.*

Proof. A common percentage transmission cost increase, s, raises each γ_θ to $\gamma_\theta (1+s)$ and so reduces each π_θ proportionately to $\frac{\pi_\theta}{1+s}$. From (9), such a common cost increase means $N(s)(1+s)$ is constant, where $N(s)$ is the equilibrium aggregate message volume under common cost increase s. Equivalently, the original $N(0)$ falls to $N(s) = \frac{N(0)}{1+s}$. Recall $\frac{n_\theta}{N} = 1 - \left(\frac{1}{\phi}\frac{N}{\pi_\theta}\right)^{\frac{1}{\phi-1}}$ from (5). Since the ratio $\frac{N}{\pi_\theta}$ (on the RHS) is unaltered by the cost increase, then so is the ratio $\frac{n_\theta}{N}$ (on the LHS). Likewise, since $\frac{N}{\pi_\theta}$ is unchanged, the price support and the cumulative price distribution remain unaltered too. Consumer welfare therefore remains unchanged, profits remain zero, and so welfare remains unchanged.

Recall the condition for a sector to be active is $(b_\theta - c_\theta) q_\theta \frac{\phi}{N} > \gamma_\theta$. With a common cost increase s, the condition becomes $(b_\theta - c_\theta) q_\theta \frac{\phi}{N(s)} > \gamma_\theta (1+s)$. However, since $N(s)(1+s) = N(0)$, the condition remains unaltered. ■

The economics of raising transmission rates are the economics of rent dissipation. Doubling the cost in each sector simply halves the number of ads sent per sector. The intuition comes from the fact that both N and n_θ are homogeneous of degree minus one. The sector choice probabilities (n_θ/N) are then homogeneous of degree zero in the percentage cost increase. The advertised price distribution, $F(p,\theta)$ is then also independent of such cost rises. This also explains why no sectors exit in the face of a common cost increase: doubling transmission costs also doubles the chance the highest priced sender makes a sale (since it faces half the competition).

We next look at sector specific cost increases.

5.1.1 Crowding out by higher transmission-cost senders?

Proposition 13 suggests that low transmission-cost sectors do not inflict more damage on high transmission-cost ones, or vice versa, at equilibrium. All sectors are in excess, but no group should be singled out.

This result leads us to ask whether a deterioration in a sector - say an increase in the sector's sending cost (like a tax with the proceeds discarded) - can reduce welfare. As we shall show, such an increase cannot help if all sectors are roughly similar, but it can if they are sufficiently asymmetric and a low-surplus sector gets worse (or even becomes inviable). From (17), the relevant welfare derivative is

$$\begin{aligned}\frac{dW}{d\gamma_\theta} &= -n_\theta + \sum\nolimits_{\theta'=1}^{\Theta} \frac{\partial W}{\partial n_{\theta'}}\frac{dn_{\theta'}}{d\gamma_\theta} + \frac{\partial W}{\partial N}\frac{dN}{d\gamma_\theta} \\ &= -n_\theta + \frac{\partial W}{\partial N}\frac{dN}{d\gamma_\theta},\end{aligned}$$

since each $\frac{\partial W}{\partial n_{\theta'}} = 0$ at equilibrium for active sectors. This expression indicates that there is a trade-off. From (21), $\frac{\partial W}{\partial N} = -\frac{1}{N}\sum\nolimits_{\theta'=1}^{\Theta} n_{\theta'}\gamma_{\theta'}$. The other desired term is $\frac{dN}{d\gamma_\theta}$. From (9), we have $\frac{dN}{d\gamma_\theta} = -N\frac{1}{\gamma_\theta}\frac{1}{\Theta}\frac{\chi_\theta}{\bar{\chi}}$, where we recall that $\bar{\chi}$ is the average value of $\chi_\theta = \left(\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}$. Pulling these expressions together, the derivative condition is:

$$\frac{dW}{d\gamma_\theta} = -n_\theta + \frac{1}{\gamma_\theta}\frac{1}{\Theta}\frac{\chi_\theta}{\bar{\chi}}\sum\nolimits_{\theta'=1}^{\Theta} n_{\theta'}\gamma_{\theta'}.$$

Under symmetry, $dW/d\gamma_\theta = 0$. This means that a rise in one sector's transmission costs has no effect at the margin. To deal with asymmetric cases, it helps to rewrite the above expression as

$$\begin{aligned}\frac{dW}{d\gamma_\theta} &\overset{s}{=} -\frac{n_\theta\gamma_\theta\Theta}{\sum\nolimits_{\theta'=1}^{\Theta} n_{\theta'}\gamma_{\theta'}} + \frac{\chi_\theta}{\bar{\chi}} \\ &= -\frac{\Gamma_\theta}{\bar{\Gamma}} + \frac{\chi_\theta}{\bar{\chi}},\end{aligned}$$

where the symbol $\overset{s}{=}$ denotes that the derivative has the sign of the expression, and where $\Gamma_\theta = n_\theta\gamma_\theta$ is the aggregate transmission cost for sector θ, and $\bar{\Gamma}$ is the average of these. A marginal sector has Γ_θ close to zero because it delivers few messages: ceteris paribus, if the sector has higher message costs than average, then its π_θ is low, so its χ_θ is high. Therefore, for a marginal sector, the second term dominates: a weak (high transmission cost) sector's rise in costs (which effectively can bring about its demise) is socially beneficial:

Proposition 15 *Welfare rises when transmission costs increase in weak sectors with high transmission costs.*

The analysis of this sub-section indicates the weak products with high transmission costs as being socially harmful. This holds despite them having a small foothold: one might have otherwise suspected strong transmission cost (high-profit) products because they are responsible for the most crowding. We take a different perspective in the next subsection, by pointing the finger at low-cost products as being over-represented, when all messages are scaled back proportionately by a proportional tax.

5.2 Taxing transmission

Proposition 12 suggests that taxing transmission will raise welfare. The next result clarifies.

Proposition 16 *A uniform percentage transmission tax increases welfare, as does a tax on a weak sector with high transmission costs.*

Proof. These results are simple corollaries of the last two Propositions. With a common percentage tax (at rate $\tau = s$), Proposition 14 shows that consumer welfare remains unchanged, and profits remain zero. Hence, welfare rises by the amount of tax raised. A similar observation applies to Proposition 15. ■

Percentage (across the board) taxes have no effect on total transmission costs borne by senders, due to adjustment to the zero profit equilibrium. If tax revenues were discarded, a tax would have no effect on welfare. Any tax not lost in the collection is therefore a social gain, and gets transferred purely from costs. Since profits are zero, consumers are just as well off since they face the same situation (same distributions, but fewer overall messages). The tax is therefore raised *without deadweight loss*. However, Proposition 14 suggests that there may be welfare gains from discriminatory taxation. This leads us to investigate the optimality of the allocation induced by uniform taxes.

5.2.1 Crowding out by low transmission-cost senders?

Proposition 13 showed that the base allocation of messages was optimal for the equilibrium message volume, N^e. By Proposition 16, an equal percentage tax on transmission scales back messages proportionately. However, unless transmission costs are the same, it may be that the scaled-back message levels induced by a non-negligible tax are not optimal for the new (given) total volume of messages. Indeed, the proof of Lemma 11 gives the partial welfare derivative (19)

$$\frac{\partial W(n,N)}{\partial n_\theta} = (b_\theta - c_\theta)\, q_\theta \mathbb{P}_\theta - \gamma_\theta,$$

and this expression still holds in the presence of a tax (although the arguments in $\mathbb{P}_\theta$ are proportionately smaller). These partial derivatives are still to be equalized across sectors at any constrained optimal allocation for given N^e. However, the market equilibrium condition in the presence of a proportional tax on transmission becomes $(b_\theta - c_\theta)\, q_\theta \mathbb{P}_\theta = \gamma_\theta (1+\tau)$. Substituting,[34]

$$\frac{\partial W(n^e,N^e)}{\partial n_\theta} = \tau \gamma_\theta. \tag{23}$$

This means that the allocation is constrained optimal (all the $\partial W(n^e,N^e)/\partial n_\theta = 0$) either if $\tau = 0$ (where we evaluated the earlier welfare derivative), or if all the transmission costs, γ_θ, are equal. Otherwise, ramping up the transmission cost with a tax causes an allocative distortion: from (23), the higher-cost messages ought to be provided more (and the lower-cost ones less). This means that the cheaper messages tend to be overused in equilibrium (in the presence of the tax). These are the ones associated with the most dissipation, ceteris paribus. In summary:

Proposition 17 *Given a positive uniform percentage transmission tax, the market allocation overprovides messages from low transmission cost sectors, in both relative and absolute terms.*

This suggests that the low transmission-cost sectors are over-represented in the population of messages

[34] Loosely, this is akin to an envelope result: here is the revenue raised on the last message in the sector when sector sizes are optimal.

(in the sense that they ought to be scaled back more than proportionately).[35] Although the proportional tax does not effect choice probabilities, the fact that the allocation is no longer optimal if transmission costs are different means that the optimal tax (given a target N) is not a proportional one. The results above instead suggest that the optimal tax should fall more heavily on the cheaper message communications: from (23), the sector-specific tax rate that ensures all sectors have the same marginal social benefit entails τ_θ inversely proportional to γ_θ.[36]

While Proposition 15 suggests that high transmission cost sectors are the main source of distortions, Proposition 17 suggests that low transmission cost sectors are more harmful. However, these results have taken different perspectives on the "blame" issue. The first result shows that a cost increase may be socially beneficial, even without revenue, and reflects the idea that sectors with low social value just take up room in the message space. The second result shows that there are relatively too many messages from low-cost sectors, given a number of messages, and reflects the idea that such sectors are responsible for excessively crowding the message space. In this context, the low-cost sectors are also the sectors with small tax raised per message, in this sense the high-cost sectors have the additional social benefit of a larger revenue per message.

6 Extensions

6.1 Distractions

Several of the strong properties in the normative analysis above relied critically on the homogeneity property of the numbers of messages sent. One natural way to relax this property is to introduce another source of competition for attention.

Think of consumers as having a limited amount of time, or a limited attention span. They cannot process all the information coming at them. Jostling with the price of MicroSoft Word or a supermarket flyer for

[35] If half the messages were discarded across the board, then the marginal benefit of a message in a sector would be (retaining the original number values as arguments): $\frac{\partial W(n,N/2)}{\partial n_\theta} = (b_\theta - c_\theta)\, q_\theta \frac{2\phi}{N}\left(1 - \frac{n_\theta}{N}\right)^{\phi-1} - \gamma_\theta$. Since $(b_\theta - c_\theta)\, q_\theta \frac{\phi}{N}\left(1 - \frac{n_\theta}{N}\right)^{\phi-1} = \gamma_\theta$ at the original equilibrium, then $\frac{\partial W(n,N/2)}{\partial n_\theta} = \gamma_\theta$. This also indicates a benefit from scaling back the low-cost end relatively more.

[36] Indeed, the first-best optimum entails just one message per sector, which also suggests more than proportional scaling back through taxes of low-cost sectors.

pork chops is a really important email from a Dean or a crying child. We model this outside competition for attention as further "distractions" to attention. Formally, this means there are n_0 other messages (or activities) which compete for attention along with the messages sent from the advertising sectors. Hence now we have $N = \sum_{\theta=0}^{\bar{\Theta}} n_\theta$. We will further associate an exogenous social value π_0 to each message (or activity) examined from the outside sector, and we assume that this value accrues on *each* such message examined.

This amendment relaxes some of the stronger properties of the equilibrium configuration, but retains other key ones, most notably that the market equilibrium is still optimal given N^e. However, raising τ in all sectors (except the distraction sector, which in that sense can be viewed as an untaxed sector) no longer causes the price spread to remain unchanged for all sectors. The key property before was that $N(\tau)/(1+\tau)\pi_\theta$ was independent of τ. Now that is no longer true, and some sectors get evicted as tax rates rise.

6.1.1 Message volume with distractions

With distractions, it is still true that each sector's message share is $\frac{n_\theta}{N} = 1 - \left(\frac{N}{\phi}\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}$, $\theta = 1, ..., \bar{\Theta}$ (see (5)). However, to find the total number of messages, N, now means adding in the outside sector, so the earlier CES form is amended to yield the implicit form:

$$N = n_0 + N \sum_{\theta=1}^{\bar{\Theta}} \left(1 - \left(\frac{N}{\phi}\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}\right).$$

Writing this out, we have

$$n_0 + N\left(\bar{\Theta} - 1\right) = N^{\frac{\phi}{\phi-1}} \sum_{\theta=1}^{\bar{\Theta}} \left(\frac{1}{\phi}\frac{1}{\pi_\theta}\right)^{\frac{1}{\phi-1}}. \tag{24}$$

The LHS is linear in N (with a positive intercept), and the RHS is convex (and starts at 0), so that there is one and only one intersection with $N > 0$. Hence there is a unique solution $N > 0$. The comparative static properties of the equilibrium are quite simple. For example, a higher value of n_0 leads to a lower N, and n_θ falls in all other sectors.

6.1.2 Welfare analysis

We first show that there is still the right allocation of N, but too many messages. The welfare function is now written as

$$W = \sum\nolimits_{\theta=1}^{\bar{\Theta}} \left[(b_\theta - c_\theta) \, q_\theta \mathbb{Q}_\theta - \gamma_\theta n(\theta) \right] + n_0 \pi_0 \frac{\phi}{N},$$

where π_0 denotes the net social benefit per distraction, and n_0 distractions vie for the attention span of ϕ given N total competitors. The result of Proposition 13 with $n_0 > 0$ still holds true: with a distraction, the equilibrium allocation is still constrained optimal.

The proof follows the lines of the earlier one: again, for the active message-sending sectors, all the marginal benefits are equalized, $1...\bar{\Theta}$. For any given N, the partial derivative marginal benefit expressions (which are to be equalized across all sectors in the second-best problem of choosing the optimal allocation of N messages) are the same as those given before, and hence the equilibrium still has the "right" allocation of the messages across the $\bar{\Theta}$ sectors.

Now consider a uniform percentage tax on all sectors, except the "untaxed" sector, n_0. From the welfare function above, the effect of a tax is $\frac{dW}{d\tau} = \sum\nolimits_{\theta=1}^{\bar{\Theta}} \frac{\partial W}{\partial n_\theta} \frac{dn_\theta}{d\tau} + \frac{\partial W}{\partial N} \frac{dN}{d\tau}$.[37] Evaluating at $\tau = 0$ yields again the result that the equilibrium entails the optimal allocation, $\frac{\partial W}{\partial n_\theta} = 0$, where the zero comes from the zero profit condition, as seen before. Hence, $\frac{dW}{d\tau} = \frac{\partial W}{\partial N} \frac{dN}{d\tau}$, and we know $\frac{dN}{d\tau} < 0$. Also, $\frac{\partial W}{\partial N} < 0$ since each $\mathbb{Q}_\theta$ term is decreasing in N and the additional term, $n_0 \pi_0 \frac{\phi}{N}$, is decreasing in N (given that $\pi_0 > 0$). Hence welfare increases locally from a uniform percentage tax, and with distractions, a tax has the additional benefit of rendering more prominent the "distractions."

6.2 Elastic demand

So far we have supposed that demand is rectangular. We now argue that the positive analysis remains tractable when we replace this assumption by a downward-sloping demand curve. We can still fully determine the shape of the equilibrium price distribution when there are many sectors, each with a specific conditional

[37] To find $N(\tau)$, and hence $n_\theta(\tau)$, use the previous expressions and note that $n_0 + N(\Theta - 1) = N^{\frac{\phi}{\phi-1}} (1+\tau)^{\frac{1}{\phi-1}} \sum\nolimits_{\theta=1}^{\bar{\Theta}} \left(\left(\frac{1}{\phi} \frac{1}{\pi_\theta} \right)^{\frac{1}{\phi-1}} \right)$, so that the message total goes down with τ.

demand function. The key property is that we can still back out the number of messages in the sector from the calculus for the highest-priced firm, and thence determine the entire price distribution, Details are below. Surprisingly, we also retain the key property that a percentage tax on message transmission does not change the distribution, but simply scales back the number of messages proportionately.

Suppose then that sector θ is associated to a conditional demand, $q_\theta(p)$, with the understanding that the consumer will buy this number of units at the lowest price, p, held. Assume that demand begets a quasi-concave profit function with a maximizing price $\hat{p}_\theta$. The corresponding conditional (on being the only one found from the sector) profit is $(\hat{p}_\theta - c_\theta) q_\theta(\hat{p}_\theta)$, and so the profit per dollar transmission cost is $\hat{\pi}_\theta = \frac{(\hat{p}_\theta - c_\theta) q_\theta(\hat{p}_\theta)}{\gamma_\theta}$, which therefore plays exactly the same role as did π_θ in the earlier analysis with rectangular demand. In equilibrium, no firm will charge more than $\hat{p}_\theta$ because profits can be increased by charging $\hat{p}_\theta$.

The parallel analysis to that above yields the equilibrium price distribution as

$$F(p,\theta) = \frac{1 - \left(\frac{N}{\phi\hat{\pi}_\theta}\right)^{\frac{1}{\phi-1}} \left(\frac{(\hat{p}_\theta - c_\theta) q_\theta(\hat{p}_\theta)}{(p - c_\theta) q_\theta(p)}\right)^{\frac{1}{\phi-1}}}{1 - \left(\frac{N}{\phi\hat{\pi}_\theta}\right)^{\frac{1}{\phi-1}}}$$

(cf. (14)), where $N = \phi \frac{(\bar{\Theta}-1)^{\phi-1}}{\left(\sum_{\theta=1}^{\bar{\Theta}} \left(\frac{1}{\hat{\pi}_\theta}\right)^{\frac{1}{\phi-1}}\right)^{\phi-1}}$ (which is the same expression as (9) except with $\hat{\pi}_\theta$ replacing π_θ). Now $\underline{p_\theta}$ is given implicitly by $\left(\underline{p_\theta} - c_\theta\right) q_\theta\left(\underline{p_\theta}\right) = \left(\frac{N}{\phi}\right)\gamma_\theta$ (cf. (15)), which has a unique price solution for $\underline{p_\theta} < \hat{p}_\theta$ under the assumption that profit is strictly quasi-concave.[38] Compared to the earlier distribution for rectangular demand, if we set $\hat{p}_\theta = b_\theta$, the distribution is now stochastically lower (FOSD) because lower prices are relatively more attractive than before because of the demand expansion effect.

It is clear that the price distribution above is independent of a percentage tax on message transmission costs if N is proportional to these costs. But this is true by the linear homogeneity property of (9).

Proposition 13 addressed the optimal allocation of a given number of messages. In the earlier context, the price distribution within a sector is irrelevant for total surplus (though not for surplus distribution). Now price levels matter. We could, of course, assume first-best pricing at marginal cost, but this would

[38] If the profit function is not quasiconcave, the support of the price distribution will have a gap for any price such that profit is no lower at a lower price.

scarcely reflect the market situation. We can scarcely assume either that the equilibrium price distribution is given and then vary message numbers by sector since this would violate the assumed equilibrium zero-profit condition. This means that we cannot meaningfully perform a similar exercise to the earlier one.

7 Conclusions

The Information Age is characterized by a surfeit of information sent at relatively low cost. Modern economies involve many media which can be used to catch the attention of prospective consumers, so the attention span of consumers is likely larger than ever before. Yet modern economies also involve many product classes. These factors interact to determine the degree of competitiveness of sectors, as reflected in the degree of price dispersion. Below we bring together some of the key comparative static properties and how they are transmitted.

First, new product classes may displace others by crowding information spans. As profitable new opportunities arise, or indeed, as the cost of communicating them through new media falls, less profitable classes are displaced. Total information volume rises, and new (or improved) sectors carve out advertising market shares at the expense of the others. Nevertheless, sufficiently strong other sectors may see a rise in their absolute message volume because crowding relaxes price competition leading to stochastically higher prices. This can encourage messages when the enhanced profit effect dominates the direct crowding effect.

Second, ceteris paribus, increasing the number of product classes causes an initial acceleration in the volume of messages as crowding raises prices making more ads profitable. Eventually this tails off, in a classic S-shaped (logistic) volume relation over time, with an upper bound to message volume.

Third, as consumer attention rises through new outlets reaching consumers, prices fall stochastically as competition is enhanced. This gives rise to the Information Hump: information volume initially rises as it becomes easier to get messages across. But the lower prices eventually come to dominate as it becomes less profitable to send messages as it is likely that other offers register with the consumer. This suggests that both more attention and more product classes raise the volume of information. Eventually though the attention span effect reduces information volume and increases competition. Thus, whether prices get lower depends crucially on whether attention rises "faster" than the range of (desirable) goods.

The model borrows heavily from Butters (1977) in using a zero-profit condition to derive equilibrium price distributions. But it differs in key respects in assumptions and conclusions. While Butters' model assumes that each message is read by some consumer, here some messages are "lost" because they are not read at all. We stress too the competition for attention across sectors, which gives rise to cross-sector effects in pricing and message volume. While Butters finds that the overall level of advertising is optimal, we have too much advertising, though a constrained optimality result is retained in the sense that the allocation across sectors is optimal, given the equilibrium message level.

The intuition for our optimal allocation of ads across sectors, given the total (excessive) volume, is as follows. First, the congestion externality of the overall ad level is the same regardless of which sector sends an extra ad (the term $\partial W/\partial N$ in the normative analysis). Second, the individual sector contribution to welfare from an extra ad is the probability it is seen, weighted by its social contribution, from which is subtracted the sending cost. As with the Butters model, this is the profit of the top firm, and so is zero for all sectors.

The model delivers a detailed picture of equilibrium price distributions across asymmetric sectors competing for attention. Equilibrium message ratios are shown to obey an *inverse* IIA property. The equilibrium total volume of advertising messages is a CES function of the individual sectors' profitability measures. This constitutes a novel derivation of such a CES function, and is instrumental in being able to derive sharp predictions.

A CES form is still central when we allow for "distractions" to the attention paid to ads. This device relaxes the homogeneity property that proportional decreases in communication costs raise ad levels proportionately, and gives rise to a modified CES form for ad levels, whereby lower costs across the board now may cause weaker sectors to exit. However, a tax on ads still raises welfare despite the introduction of an "untaxed" sector (there is still over-advertising), and the allocation of ads across sector is still optimal under the constraint of the equilibrium total volume of messages.

Some caveats to the analysis constitute further extensions. The model is one of firms seeking (passive) consumers through ads, which can be thought of as the pure Couch Potato model. The converse case has consumers seeking opportunities through search. Indeed, both sides can be active, as in Baye and Morgan (2001). One step in this direction is to allow the attention span to be endogenously determined by equating the expected surplus from an extra ad to the marginal cost of paying more attention: the current specification can be viewed as a simple version of this with prohibitive marginal cost at ϕ.

The model also views all media as equally delivering messages for attention, and is not immediately equipped to deal with which messages might be better suited to which media. Nor indeed is media pricing of message delivery given much shrift, though this is the topic of the (platform) economics of broadcasting. Instead, perhaps like billboards, web-sites and bulk mail, access price is exogenous. The crucial marketing dimension of targeting of messages to consumers (for example through the use of specific media) has been closed down through the device of a single representative consumer. Likewise, messages are assumed to be sampled randomly, so there is no allowance for the consumer to pay more attention to particular message types. The Economics of Attention has yet to be fully fleshed out in these broader directions.

8 Appendix

8.1 Comparison to Butters model

Butters (1977) supposes M consumers, and a *single* sending sector (so we can suppress the subscript θ in what follows). Letters are sent randomly, and each message reaches only a *single* consumer (ours potentially reach all consumers). Consumers examine *all* the messages received, and each buys at the lowest price received. As with our model, the equilibrium price support has no atoms, no holes, and runs up to b. It starts at $c + \gamma$, because a message at that price is surely read by whoever receives it, and it is a winner (in our model, it must start higher because even the best deal may be unread).

We follow Butters in equating the probability of a sale from two different perspectives. The first is the zero profit condition, $\mathbb{P} = \frac{\gamma}{p-c}$. The second is the finding probability for the price p. For the price b, the likelihood of finding an empty letter box (the only way for the highest price to make a sale) must therefore equal $\frac{\gamma}{b-c}$. This is thus the fraction of the market unserved, and so is a key statistic in comparing equilibrium to optimum.

The corresponding welfare function is $W = (b - c)M\Lambda - \gamma N$ if N messages are sent, where Λ is the fraction of consumers informed. Hence the optimal number of ads is determined from $(b-c)\,M\frac{d\Lambda}{dN} - \gamma$: this equation suggests that an exponential form for the probability of finding an empty letterbox will give equivalence with the equilibrium. This remark underscores the formulation of Butters' letterbox technology. To derive this, note that the probability that at least one of N letters sent reaches a particular one of the M letterboxes is $1 - \left(1 - \frac{1}{M}\right)^N$. When M is large, this is approximately $1 - \exp(-N/M)$ $(= \Lambda)$. Hence, $\frac{d\Lambda}{dN} = \frac{1}{M}\exp(-N/M) = \frac{1}{M}(1 - \Lambda)$, from which it follows that the number of uninformed at the optimum is $\frac{M\gamma}{b-c}$, the same as in equilibrium.[39]

Finally, consider the equilibrium advertised price distribution in the Butters model. Let the number of letters priced below p be $A(p)$ (which therefore replaces N in the logic of the previous paragraph). Hence the probability of a letter missing all lower-priced letters in a mailbox is $\exp(-A(p)/M)$ which must equal $\frac{\gamma}{p-c}$ by the zero profit condition. The form of $A(p)$ and its properties (decreasing, concave) follow directly.

8.2 Sales price distribution

The advertised price distribution is $F(p,\theta)$ as given in Proposition 8; denote the corresponding density by $f(p,\theta)$. Then the sales price density at p is the advertised price density, $f(p,\theta)$, times the probability, $\mathbb{P}(p,\theta)$, that the advertised price makes a sale (up to a multiplicative constant, k_1). Since the latter is $\frac{\gamma_\theta}{p-c_\theta}$, this means that the sales price density is:

$$g(p,\theta) = k_1 f(p,\theta)\,\mathbb{P}(p,\theta) = k_2\left(\frac{1}{p-c_\theta}\right)^{\frac{1}{\phi-1}+2},$$

where k_2 is a constant to be determined. The corresponding cumulative distribution of the sales price (cf. (14)) is

$$G(p,\theta) = k_3\left[\left(\frac{1}{\underline{p}-c_\theta}\right)^{\frac{1}{\phi-1}+1} - \left(\frac{1}{p-c_\theta}\right)^{\frac{1}{\phi-1}+1}\right].$$

The constant k_3 can be determined from the relation $G(b_\theta,\theta) = 1$. Differentiating gives the following properties (cf. Proposition 8).

[39] The interpretation is that the business stealing and consumer surplus appropriation externalities net out.

The equilibrium sales price density in sector θ is decreasing and convex on $[\underline{p_\theta}, b_\theta]$, with cumulative distribution given by

$$G(p,\theta) = \frac{\left(\frac{1}{\underline{p_\theta}-c_\theta}\right)^{\frac{1}{\phi-1}+1} - \left(\frac{1}{p-c_\theta}\right)^{\frac{1}{\phi-1}+1}}{\left(\frac{1}{\underline{p_\theta}-c_\theta}\right)^{\frac{1}{\phi-1}+1} - \left(\frac{1}{b_\theta-c_\theta}\right)^{\frac{1}{\phi-1}+1}}, \tag{25}$$

where $\underline{p_\theta}$ is given by (15).

This is the distribution of the actual transaction prices for sector θ, i.e., conditional on a sale being made. Since the probability of a sale being made in sector θ is $\mathbb{Q}_\theta = 1 - \left(1 - \frac{n_\theta}{N}\right)^\phi$, then $\mathbb{Q}_\theta G(p,\theta)$ represents the (unconditional) probability of a sale being made (or, indeed, the fraction of consumers buying) in sector θ at a price below p. This statistic allows us to calculate the expected consumer surplus from the sector.[40]

The next Figure shows the difference: the advertised price distribution, $F(p,\theta)$ is given as the solid line. (The parameter values used are: $b = 1$, $c = 0$, $N/\phi = 10$, $\phi = 10$ and $\gamma = 0.025$).[41] The distribution of actual transactions conditional on a sale being made, $G(p,\theta)$, is the dotted line on top. The dashes represent the cumulative distribution of actual sales in the population, $\mathbb{Q}_\theta G(p,\theta)$, for sector θ. This sales price distribution lies above the advertised price distribution for low p. This means simply that more sales are made at low prices (as high priced ads are beaten). This must happen throughout the whole range of prices in Butters' model because each ad is received by someone. However, in our context, there is a probability that ads are not received at all. This feature is reflected in the fact that $\mathbb{Q}_\theta G(b) < 1$ in our model (see Figure 3): the consumer may get no ads from the sector and no sale is made at all.

[40] We can use $G(p,\theta)$ to calculate the *size distribution of firms* within sector θ. In particular, we can simply replace $p = c_\theta + \frac{\gamma_\theta}{Q}$ (from the zero profit condition) where $Q = \mathbb{Q}q_\theta$ is the size of the firm, in terms of units sold. Substituting in (25) gives:

$$H(Q,\theta) = 1 - G\left(c_\theta + \frac{\gamma_\theta}{Q}, \theta\right) = \left[\frac{\left(\frac{Q}{\gamma_\theta}\right)^{\frac{1}{\phi-1}+1} - \left(\frac{Q_L}{\gamma_\theta}\right)^{\frac{1}{\phi-1}+1}}{\left(\frac{Q_H}{\gamma_\theta}\right)^{\frac{1}{\phi-1}+1} - \left(\frac{Q_L}{\gamma_\theta}\right)^{\frac{1}{\phi-1}+1}}\right],$$

where Q_H $(= \frac{\gamma_\theta}{\underline{p_\theta}-c_\theta})$ is the highest output (corresponding to the lowest price, $\underline{p_\theta}$) and Q_L $(= \frac{\gamma_\theta}{b_\theta-c_\theta})$ is the lowest output (corresponding to the highest price, b_θ). The corresponding density, $h(Q,\theta)$, is proportional to $Q^{\frac{1}{\phi-1}}$ and so is increasing, and concave for $\phi > 2$. Note that aggregate congestion interactions DO enter here (and are simply described through the N effect), since $\underline{p_\theta} = c_\theta + \left(\frac{N}{\phi}\right)\frac{\gamma_\theta}{q_\theta}$ enters into Q_H $(= \frac{\gamma_\theta}{\underline{p_\theta}-c_\theta})$.

[41] The value $N/\phi = 10$ can be consistent with a specific number of other sectors.

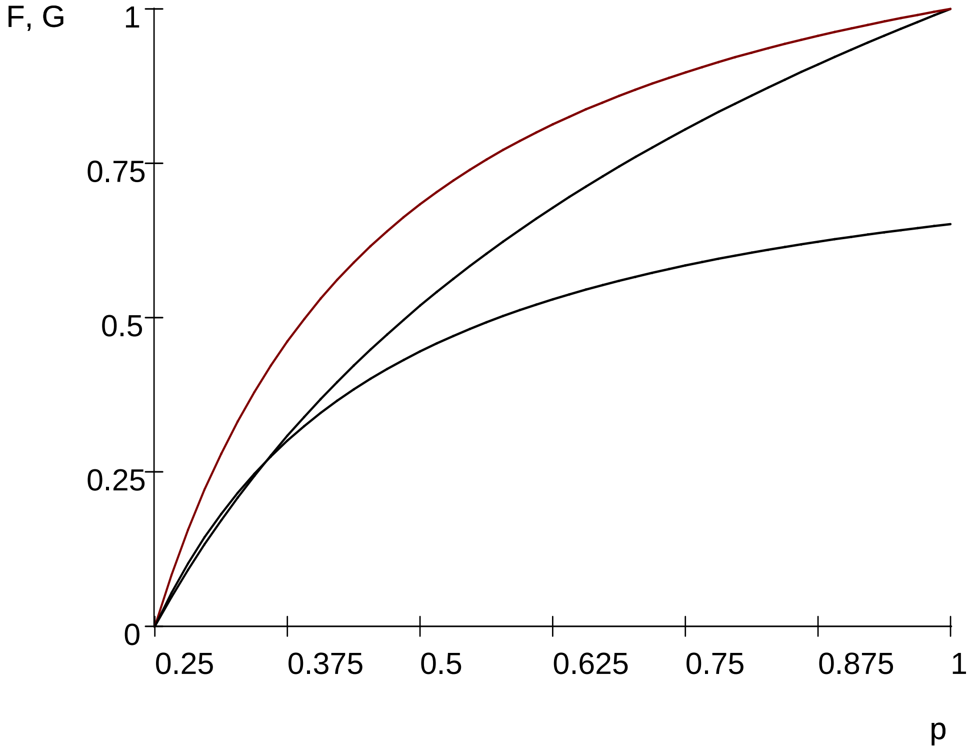

Figure 3. Advertised price functions (solid) and sales price distributions.

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